Economics 413: Economic Forecast & Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm I – Answers

1. (a) Stationary – white noise
   (b) Stationary – white noise
   (c) Stationary – all MA models are stationary
   (d) Stationary – all MA models are stationary
   (e) Stationary – all MA models are stationary
   (f) Stationary – all MA models are stationary
   (g) Stationary – $|\phi| = 0.4 < 1$ implies that the AR(1) is stationary
   (h) Nonstationary – $|\phi| = 1.5 > 1$ implies that the AR(1) is nonstationary
   (i) Stationary – $|\phi_1| + |\phi_2| = 0.45 + 0.25 < 1$ implies that the AR(2) is stationary
   (j) Nonstationary – $\phi_1 + \phi_2 + \phi_3 = 1$ implies that there is a unit root in the AR(3) model
   (k) Stationary – $|\phi_1| + |\phi_2| = 0.45 + 0.25 < 1$ implies that the ARMA(2,2) is stationary

2. (a) $MA\left(3\right)$
   (b) Tri-annual or trimester (three times per year)
   (c) $E\left(y_t\right) = E\left(\mu + \varepsilon_t + \theta_3\varepsilon_{t-3}\right) = \mu$
   (d) $\gamma_0 = E\left[\left(y_t - \mu\right)^2\right] = E\left[\left(\varepsilon_t + \theta_3\varepsilon_{t-3}\right)^2\right] = E\left(\varepsilon_t^2 + 2\theta_3\varepsilon_t\varepsilon_{t-3} + \theta_3^2\varepsilon_{t-3}^2\right) = \sigma^2\left(1 + \theta_3^2\right)$
   (e) $\gamma_1 = E\left[\left(y_t - \mu\right)\left(y_{t-1} - \mu\right)\right] = E\left[\left(\varepsilon_t + \theta_3\varepsilon_{t-3}\right)\left(\varepsilon_{t-1} + \theta_3\varepsilon_{t-4}\right)\right] = 0$, $\gamma_2 = E\left[\left(y_t - \mu\right)\left(y_{t-2} - \mu\right)\right] = E\left[\left(\varepsilon_t + \theta_3\varepsilon_{t-3}\right)\left(\varepsilon_{t-2} + \theta_3\varepsilon_{t-5}\right)\right] = 0$, $\gamma_3 = E\left[\left(y_t - \mu\right)\left(y_{t-3} - \mu\right)\right] = E\left[\left(\varepsilon_t + \theta_3\varepsilon_{t-3}\right)\left(\varepsilon_{t-3} + \theta_2\varepsilon_{t-6}\right)\right] = \theta_3\sigma^2$, $\gamma_j = E\left[\left(y_t - \mu\right)\left(y_{t-j} - \mu\right)\right] = E\left[\left(\varepsilon_t + \theta_2\varepsilon_{t-2}\right)\left(\varepsilon_{t-j} + \theta_2\varepsilon_{t-j-1}\right)\right] = 0\ \forall j > 3$
   (f) $\rho_1 = \frac{\gamma_1}{\gamma_0} = 0$, $\rho_2 = \frac{\gamma_2}{\gamma_0} = 0$, $\rho_3 = \frac{\gamma_3}{\gamma_0} = \frac{\theta_3\sigma^2}{\sigma^2\left(1+\theta_3^2\right)} = \frac{\theta_3}{\left(1+\theta_3^2\right)}$, $\rho_j = \frac{\gamma_j}{\gamma_0} = 0\ \forall j > 3$. The ACF has a spike equal to $\frac{\theta_3}{\left(1+\theta_3^2\right)}$ at lag 3 and all other lags are equal to zero.
   (g) $|\theta_3| < 1$
   (h) $y_t = \mu + \left(1 + \theta_3 L\right)\varepsilon_t \Rightarrow \frac{y_t}{\left(1+\theta_3 L\right)} = \frac{\mu}{\left(1+\theta_3\right)} + \varepsilon_t$ which is an $AR\left(\infty\right)$. Alternatively, if we solve for $\varepsilon_t = y_t - \mu - \theta_3\varepsilon_{t-3}$ and then substitute in for $\varepsilon_{t-3} = y_{t-3} - \mu - \theta_3\varepsilon_{t-6}$ and so on we will get $y_t = \mu + \varepsilon_t + \theta_3\left(y_{t-3} - \mu - \theta_3\left(y_{t-6} - \mu - \theta_3\left(y_{t-9} - \mu - \theta_3\left(\cdots\right)\right)\right)\right)$ which is the equivalent $AR\left(\infty\right)$.

3. (a) The ACF has a single spike at lag 1 and the height of the spike is $\widehat{\theta}/\left(1 + \widehat{\theta}^2\right) = 0.485/\left(1 + 0.485^2\right)$. The PACF has an infinite number of (positive) spikes.
   (b) $y_t = 0.160 + 0.485\widehat{\varepsilon}_{t-1} + \widehat{\varepsilon}_t$ or we could write it as $\widehat{y}_t = 0.160 + 0.485\widehat{\varepsilon}_{t-1}$
   (c) $\widehat{\mu} = 0.16$, $\widehat{\gamma}_0 = \left(1 + \widehat{\theta}^2\right)\widehat{\sigma}^2 = \left(1 + 0.485^2\right)\left(4.449\right)^2$, $\widehat{\gamma}_1 = \widehat{\theta}\widehat{\sigma}^2 = 0.485\left(4.449\right)^2$, $\gamma_j = 0\forall j > 1$