Economics 413: Economic Forecast & Analysis

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Final – Answers

1. (a) Random walk - AR(1)

   (b) $E(Y_t) = E(Y_{t-1} + \varepsilon_t)$ using backward substitution we can see that $E(Y_t) = Y_0$

   (c) We know that this process is non-stationary because $\phi = 1$

      i. A. $\Delta Y_t = \varepsilon_t$

         B. $E(\Delta Y_t) = E(\varepsilon_t) = 0$

         C. $V(\Delta Y_t) = V(\varepsilon_t) = \sigma^2$, $COV(\Delta Y_t, \Delta Y_{t-j}) = E(\varepsilon_t \varepsilon_{t-j})\, \forall j \neq 0$

2. (a) $\phi_1 + \phi_2 = 1$ implies that there is a unit root

   (b) We should take a first difference of the series

   (c) $y_t = a_0 + a_1 t + a_2 t^2 + z_t$

   (d) $\sum_{t=1}^{T} \widehat{z}_t^2 = \sum_{t=1}^{T} \left(y_t - \widehat{a}_0 - \widehat{a}_1 t - \widehat{a}_2 t^2\right)^2$

   (e) $\ln \mathcal{L}\left(\phi_1, \phi_2, \theta_1, \theta_2, \sigma^2\right) = -\frac{T}{2} \ln \sigma^2 - \frac{T}{2} \ln 2\pi - \frac{1}{2\sigma^2} \sum_{t=1}^{T} \left(\widehat{z}_t - \phi_1 \widehat{z}_{t-1} - \phi_2 \widehat{z}_{t-2} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}\right)^2$

3. $Y_t = c + \delta(u_t + \psi u_{t-1}) + \varepsilon_t = c + \delta u_t + \delta\psi u_{t-1} + \varepsilon_t$ noting that $E(Y_t) = c \equiv \mu$

   (a) $\widehat{Y}_{t+1|t} = E(Y_{t+1}|\Omega_t) = E(c + \delta u_{t+1} + \delta\psi u_t + \varepsilon_{t+1}|\Omega_t) = c + \delta\psi u_t$,
   $\widehat{Y}_{t+h|t} = E(Y_{t+h}|\Omega_t) = E(c + \delta u_{t+h} + \delta\psi u_{t+h-1} + \varepsilon_{t+h}|\Omega_t) = c\ \forall h > 1$

   (b) $e_{t+1} = Y_{t+1} - \widehat{Y}_{t+1|t} = c + \delta u_{t+1} + \delta\psi u_t + \varepsilon_{t+1} - (c + \delta\psi u_t) = \delta u_{t+1} + \varepsilon_{t+1}$,
   $e_{t+h} = Y_{t+h} - \widehat{Y}_{t+h|t} = c + \delta u_{t+h} + \delta\psi u_{t+h-1} + \varepsilon_{t++h} - (c) = \delta u_{t+h} + \delta\psi u_{t+h-1} + \varepsilon_{t++h}\ \forall h > 1$

   (c) $V(e_{t+1}) = V(\delta u_{t+1} + \varepsilon_{t+1}) = \delta^2\sigma_u^2 + \sigma_\varepsilon^2$,
   $V(e_{t+h}) = V(\delta u_{t+h} + \delta\psi u_{t+h-1} + \varepsilon_{t++h}) = \delta^2\sigma_u^2 + \delta^2\psi^2\sigma_u^2 + \sigma_\varepsilon^2\ \forall h > 1$

   (d) $P\left[\widehat{Y}_{t+1|t} - 2\sqrt{\delta^2\sigma_u^2 + \sigma_\varepsilon^2} < Y_{t+1} < \widehat{Y}_{t+1|t} + 2\sqrt{\delta^2\sigma_u^2 + \sigma_\varepsilon^2}\right] =$
   $P\left[(c + \delta\psi u_t) - 2\sqrt{\delta^2\sigma_u^2 + \sigma_\varepsilon^2} < Y_{t+1} < (c + \delta\psi u_t) + 2\sqrt{\delta^2\sigma_u^2 + \sigma_\varepsilon^2}\right] \approx 0.95$,
   $P\left[\widehat{Y}_{t+h|t} - 2\sqrt{\delta^2\sigma_u^2 + \delta^2\psi^2\sigma_u^2 + \sigma_\varepsilon^2} < Y_{t+h} < \widehat{Y}_{t+h|t} + 2\sqrt{\delta^2\sigma_u^2 + \delta^2\psi^2\sigma_u^2 + \sigma_\varepsilon^2}\right] =$
   $P\left[c - 2\sqrt{\delta^2\sigma_u^2 + \delta^2\psi^2\sigma_u^2 + \sigma_\varepsilon^2} < Y_{t+h} < c + 2\sqrt{\delta^2\sigma_u^2 + \delta^2\psi^2\sigma_u^2 + \sigma_\varepsilon^2}\right] \approx 0.95\ \forall h > 1$

   (e) A plot of Y versus t will show one value for the first period $h = 1$ and then different values for the remaining periods $h > 1$. Part (a) is a straight line equal to $c + \delta\psi u_t$ in the first period and c afterwards and the upper and lower bounds will be also equal to $c + \delta\psi u_t$ plus and minus $2\sqrt{\delta^2\sigma_x^2 + \sigma_\varepsilon^2}$ in the first period and c plus or minus $.2\sqrt{\delta^2\sigma_u^2 + \delta^2\psi^2\sigma_u^2 + \sigma_\varepsilon^2}$ afterwards.

4. (a) Given that we have an AR(1) model, we do not have information on y_{1900} and hence we must start at y_{1902}.

   (b) The table uses the (ordinary) least-squares method. The objective function is $\sum_{t=1}^{T} \widehat{\varepsilon}_t^2 = \sum_{t=1}^{T} \left(y_t - \widehat{c} - \widehat{\phi} y_{t-1} - \widehat{\delta}_1 D_{1t} - \widehat{\delta}_2 D_{2t} \right)^2$.

   (c) Note that for $t = 1901$ to 1926 $y_t = (c + \delta_1) + \phi y_{t-1} + \varepsilon_t$, for $t = 1927$ to 1974 $y_t = c + \phi y_{t-1} + \varepsilon_t$ and $t = 1975$ to 2000 $y_t = (c + \delta_2) + \phi y_{t-1} + \varepsilon_t$. Therefore $(c + \delta_1) / (1 - \phi)$, $c / (1 - \phi)$ and $(c + \delta_2) / (1 - \phi)$ for the periods 1901 to 1926, 1927 to 1974 and 1975 to 2000, respectively.

   (d) Plugging in the values from above gives us $(0.768 + 0.939) / (1 - 0.278)$, $0.768 / (1 - 0.278)$ and $(0.768 + 0.820) / (1 - 0.278)$ for the periods 1901 to 1926, 1927 to 1974 and 1975 to 2000, respectively.

   (e) Simply draw flat lines through the center of each time period with values written on the vertical axis equal to those in part (d)