Economics 413: Economic Forecast & Analysis

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Final – Answers

1. (a) $E(Y_t) = E(c + \phi Y_{t-1} + \varepsilon_t) \Rightarrow \mu = c/(1-\phi)$
   (b) $V(Y_t) = E\left[(Y_t - \mu)^2\right] \Rightarrow \gamma_0 = \sigma^2/(1-\phi)^2$
   (c) $COV(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)] \Rightarrow \gamma_j = \phi\gamma_{j-1} = \phi^j\gamma_0$
   (d) $CORR(Y_t, Y_{t-j}) = \gamma_j/\gamma_0 = \phi^j\gamma_0/\gamma_0 = \phi^j$
   (e) Given that ϕ is between negative one and zero, there is oscillating decay in the ACF
2. (a) Sample ACF: $y_t = c + \rho_j y_{t-j} + \varepsilon_t - \widehat{\rho}_j$ is the jth spike in the sample ACF
   (b) Sample PACF: $y_t = c + \phi_{1j}y_{t-1} + \phi_{2j}y_{t-2} + \cdots + \phi_{jj}y_{t-j} + \varepsilon_t - \widehat{\phi}_{jj}$ is the jth spike in the sample PACF
   (c) 2 significant spikes in the sample ACF, many decaying spikes in the sample PACF – drawing some small insignificant spikes is necessary in order to receive full credit
   (d) many decaying spikes in the sample ACF, 2 significant spikes in the sample PACF – drawing some small insignificant spikes is necessary in order to receive full credit
   (e) many large (very slowly decreasing) spikes in the sample ACF, 1 large (near unity) significant spike in the sample PACF
3. (a) $\widehat{Y}_{t+1|t} = E(Y_{t+1}|\Omega_t) = E(Y_t + \varepsilon_{t+1} + \theta_2\varepsilon_{t-1}|\Omega_t) = Y_t + \theta_2\varepsilon_{t-1}$ $\widehat{Y}_{t+2|t} = E(Y_{t+2}|\Omega_t) = E(Y_{t+1} + \varepsilon_{t+2} + \theta_2\varepsilon_t|\Omega_t) = E(Y_{t+1}|\Omega_t) + \theta_2\varepsilon_t = Y_t + \theta_2\varepsilon_{t-1} + \theta_2\varepsilon_t$ $\widehat{Y}_{t+h|t} = E(Y_{t+h}|\Omega_t) = E(Y_{t+h-1} + \varepsilon_{t+h} + \theta_2\varepsilon_{t=h-2}|\Omega_t) = E(Y_{t+h}|\Omega_t) = Y_t + \theta_2\varepsilon_{t-1} + \theta_2\varepsilon_t \ \forall h > 2$
   (b) $e_{t+1} = Y_{t+1} - \widehat{Y}_{t+1|t} = \varepsilon_{t+1}$, $e_{t+2} = Y_{t+2} - \widehat{Y}_{t+2|t} = \varepsilon_{t+1} + \varepsilon_{t+2}$, $e_{t+h} = Y_{t+h} - \widehat{Y}_{t+h|t} = \varepsilon_{t+1} + \varepsilon_{t+2} + \cdots \varepsilon_{t+h} + \theta_2(\varepsilon_{t+1} + \varepsilon_{t+2} + \cdots \varepsilon_{t+h-2}) \ \forall h > 2$
   (c) $V(e_{t+1}) = \sigma^2$, $V(e_{t+2}) = 2\sigma^2$, $V(e_{t+h}) = h\sigma^2 + (h-2)\theta^2\sigma^2 \ \forall h > 2$
   (d) $\left[\widehat{Y}_{t+1|t} \pm 1.96\sqrt{\sigma^2}\right]$, $\left[\widehat{Y}_{t+2|t} \pm 1.96\sqrt{2\sigma^2}\right]$, $\left[\widehat{Y}_{t+h|t} \pm 1.96\sqrt{h\sigma^2 + (h-2)\theta^2\sigma^2}\right] \ \forall h > 2$
   (e) All forecasts should be relatively close to the last known value of Y. The confidence bands should grow with h.
4. (a) ARMA(1,1) – $Y_t = c + \phi Y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$, ARMA(1,2) – $Y_t = c + \phi Y_{t-1} + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}$
   (b) in both cases the mean is $\mu = \frac{c}{1-\phi} = -\frac{0.104062}{1-0.530569}$ for ARMA(1,1) and $= -\frac{0.146900}{1+0.892852}$ for ARMA(1,2)
   (c) in both cases we have a single lag of Y and since we do not have data in 1964, we must start the sample at 1966 (further lags of the errors can be set to zero or replaced with residuals)

(d) both have $|\phi| < 1$ and hence are both stationary

(e) each of the model selection criteria point to the ARMA(1,2) model – R^2 and $adjR^2$ are larger while $s.e.reg$, AIC and SC are smaller