Economics 413: Economic Forecast & Analysis

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Final – Answers

1. For $t < t'$, $Y_t = c + \varepsilon_t + \theta\varepsilon_{t-1}$ and for $t \geq t'$, $Y_t = (c + \psi) + \varepsilon_t + \theta\varepsilon_{t-1}$

   (a) $E(Y_t) = c$ and $E(Y_t) = (c + \psi)$, respectively

   (b) $V(Y_t) = (1 + \theta^2)\sigma^2$ for each as the MA(1)'s only differ by means

   (c) Similary, $\gamma_1 = \theta\sigma^2$ and $\gamma_j = 0$ for all $j > 1$ for each

   (d) $h = 1$, $\widehat{Y}_{t+1|t} = E(Y_{t+1}|\Omega_t) = E[(c + \psi) + \varepsilon_{t+1} + \theta\varepsilon_t|\Omega_t] = (c + \psi) + \theta\varepsilon_t$ and $\widehat{Y}_{t+h|t} = E(Y_{t+h}|\Omega_t) = E[(c + \psi) + \varepsilon_{t+h} + \theta\varepsilon_{t+h-1}|\Omega_t] = (c + \psi)\ \forall h > 1$

   (e) $e_{t+1} = Y_{t+1} - \widehat{Y}_{t+1|t} = \varepsilon_{t+1} \Rightarrow \widehat{Y}_{t+1|t} \pm 2\sigma$ and $e_{t+h} = Y_{t+h} - \widehat{Y}_{t+h|t} = \varepsilon_{t+h} + \theta\varepsilon_{t=h-1} \Rightarrow \widehat{Y}_{t+h|t} \pm 2\sigma\sqrt{(1 + \theta^2)}\ \forall h > 1$

2. (a) ARIMA(0,1,1)

   (b) The ACF should have high persistence (lots of spikes near one) and the PACF should have a single large significant spike (near one)

   (c) The figure should look like a random walk

   (d) The point estimates should be $y_t + \widehat{\theta}\varepsilon_t$ for $h = 1, 2, 3$

   (e) The forecast intervals should be $y_t + \widehat{\theta}\varepsilon_t \pm 2\widehat{\sigma}$, $y_t + \widehat{\theta}\varepsilon_t \pm 2\widehat{\sigma}\sqrt{(2 + \theta^2)}$ and $y_t + \widehat{\theta}\varepsilon_t \pm 2\widehat{\sigma}\sqrt{(3 + \theta^2)}$, respectively

3. (a) $\mathcal{L}(\cdot) = -\frac{T}{2}\ln 2\Pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=2}^{T}\varepsilon_t^2$, where $\varepsilon_t = y_t - c - \phi y_{t-1} - \theta\varepsilon_{t-1}$ and $\varepsilon_t = y_t - c - \phi y_{t-1}$ for the true model and your model, respectively

   (b) The sample ACF and PACF should look like white noise for the true model and the sample ACF and PACF should look like an AR(1) for your model

   (c) The null in each case is that of no serial correlation. We should fail to reject the null in the true model and reject the null in your model.

   (d) The point forecasts will be off by $\widehat{\theta}\varepsilon_t$ for $h = 1$, but will be the same for $h = 2$ and 3

   (e) The variance will be too small for your model as compared to the true model and hence the forecast intervals will suggest more confidence than they should

4. (a) y, our outcome of interest (growth rate of GDP for Sierra Leone)
   (b) OLS, but the model is likely estiamted via MLE
   (c) $t = 1, 2, \ldots, T$
   (d) $\widehat{c}$ - coefficient estimate
   (e) $\widehat{\phi}$ - coefficient estimate
   (f) $\widehat{\theta}$ - coefficient estimate
   (g) $se\left(\widehat{c}\right)$ - variation in the estimate
   (h) $se\left(\widehat{\phi}\right)$ - variation in the estimate
   (i) $se\left(\widehat{\theta}\right)$ - variation in the estimate
   (j) $t = \left(\widehat{c} - 0\right)/se\left(\widehat{c}\right)$
   (k) $t = \left(\widehat{\phi} - 0\right)/se\left(\widehat{\phi}\right)$
   (l) $t = \left(\widehat{\theta} - 0\right)/se\left(\widehat{\theta}\right)$
   (m) fail to reject null ($H_0 : c = 0$) at 5% level
   (n) fail to reject null ($H_0 : \phi = 0$) at 5% level
   (o) fail to reject null ($H_0 : \theta = 0$) at 5% level
   (p) $1 - SSR/SST$
   (q) $1 - \left[SSR/\left(T-3\right)\right]/\left[SST/\left(T-1\right)\right]$
   (r) $\widehat{\sigma} = \sqrt{\frac{1}{T-3}SSR}$
   (s) $SSR = \sum_{t=1}^{T} \widehat{\varepsilon}_t^2$
   (t) $\mathcal{L}\left(\cdot\right) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\widehat{\sigma}^2 - \frac{1}{2\widehat{\sigma}^2}\sum_{t=2}^{T}\widehat{\varepsilon}_t^2$
   (u) $H_0 : \phi = \theta = 0$, $F = \frac{\left(SSR_R - SSR_U\right)/3}{SSR_U/\left(T-3\right)}$
   (v) $\overline{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$
   (w) $\widehat{\sigma}_y = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(y_t - \overline{y}\right)^2}$
   (x) $AIC = T\ln\left(SSR/T\right) + 2\left(k+1\right)$
   (y) $SC = T\ln\left(SSR/T\right) + \left(k+1\right)$