I. Time Series Plots

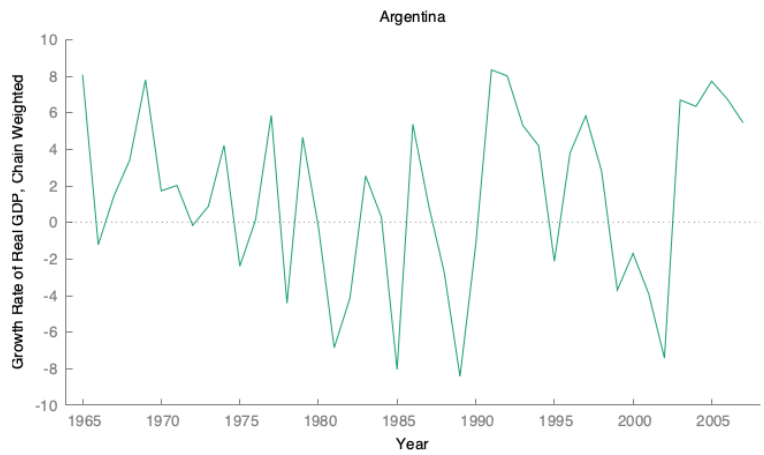
Individual Plots

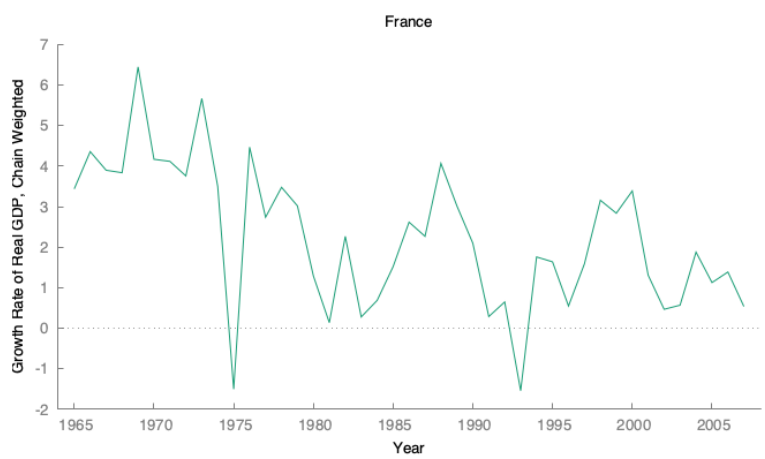
Gretl

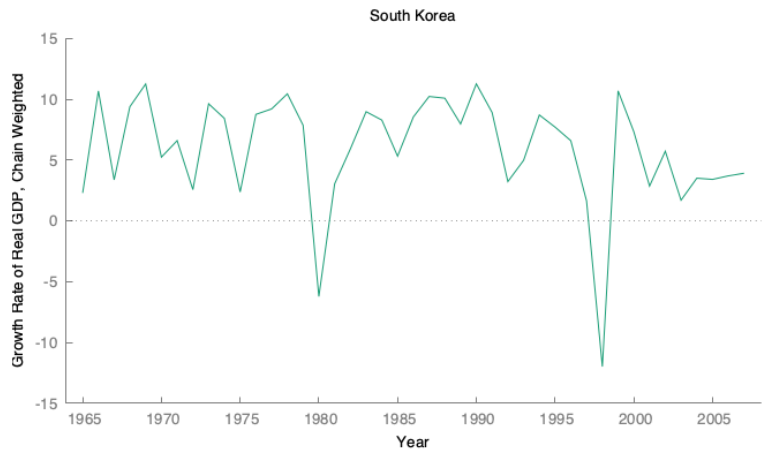
View → Graph Specified Vars → Time series plot → Click variable to plot → Green arrow

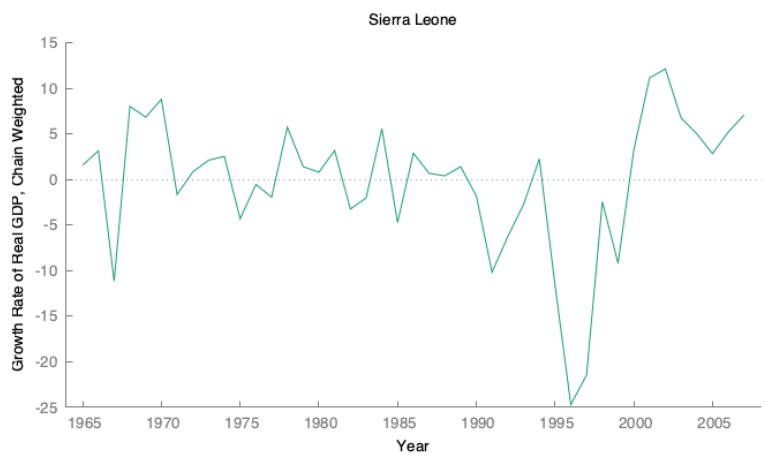
To add title:

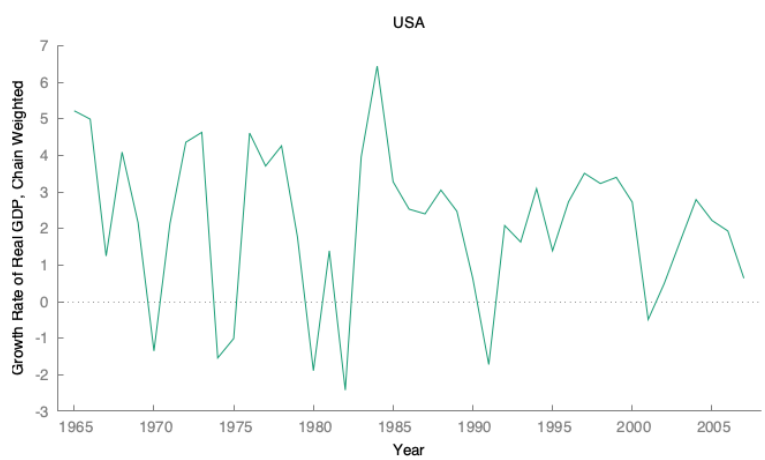
Menu (located in bottom right of generated graph window) → Edit → Title of Plot → Ok











Group Plot

Gretl

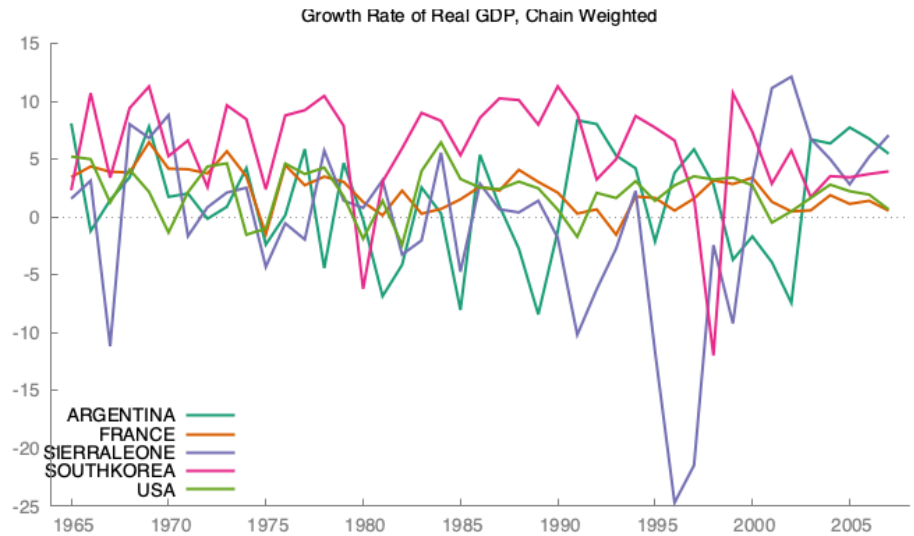
View → Graph Specified Vars → Time series plot → Highlight all variables to plot → Green arrow

To add title:

Menu (located in bottom right of generated graph window) → *Edit* → *Title of Plot* → *Ok*

To change position of legend:

Menu → *Edit* → *Key Position* → *Ok*



II. Descriptive Statistics

Sample Average

$$\boxed{\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_i}$$

Sample Standard Deviation

$$\boxed{\hat{\sigma} = \sqrt{\hat{\sigma}^2} = \left(\frac{1}{T-1}\sum_{t=1}^{T}(y_i - \bar{y})^2\right)^{1/2}}$$

Sample Covariance

$$\boxed{\hat{\sigma}_{XY} = \frac{1}{T-1}\sum_{t=1}^{T}(x_i - \bar{x})(y_i - \bar{y})}$$

Gretl

View → Summary Statistics → Highlight all variables of interest → Green Arrow → Show Full Statistics → Ok

	Mean	Median	Minimum	Maximum
ARGENTINA	1.4435	1.7300	-8.4200	8.3500
FRANCE	2.2630	2.2700	-1.5400	6.4500
SIERRALEONE	-0.20372	1.4000	-24.690	12.120
SOUTHKOREA	5.9223	6.6100	-11.970	11.280
USA	2.1488	2.4000	-2.4200	6.4400

	Std. Dev.	C.V.	Skewness	Ex. kurtosis
ARGENTINA	4.7613	3.2985	-0.38421	-0.80806
FRANCE	1.7471	0.77204	0.023830	-0.21903
SIERRALEONE	7.5207	36.917	-1.2839	2.1565
SOUTHKOREA	4.5189	0.76304	-1.7903	4.6160
USA	2.0753	0.96577	-0.45185	-0.26419

	5% perc.	95% perc.	IQ range	Missing obs.
ARGENTINA	-7.9160	8.0760	7.5800	0
FRANCE	-1.1720	5.4300	2.8100	0
SIERRALEONE	-19.496	10.662	7.7200	0
SOUTHKOREA	-4.6380	11.156	5.6000	0
USA	-1.8560	5.1740	2.2600	0

Sample Correlation Coefficient

$$\boxed{\hat{\rho}_{XY} = \frac{\hat{\sigma}_{XY}}{\hat{\sigma}_X \hat{\sigma}_Y} = \frac{\sum_{t=1}^{T}(x_i - \bar{x})(y_i - \bar{y})}{(\sum_{t=1}^{T}(x_i - \bar{x})^2)^{1/2}(\sum_{t=1}^{T}(y_i - \bar{y})^2)^{1/2}}}$$

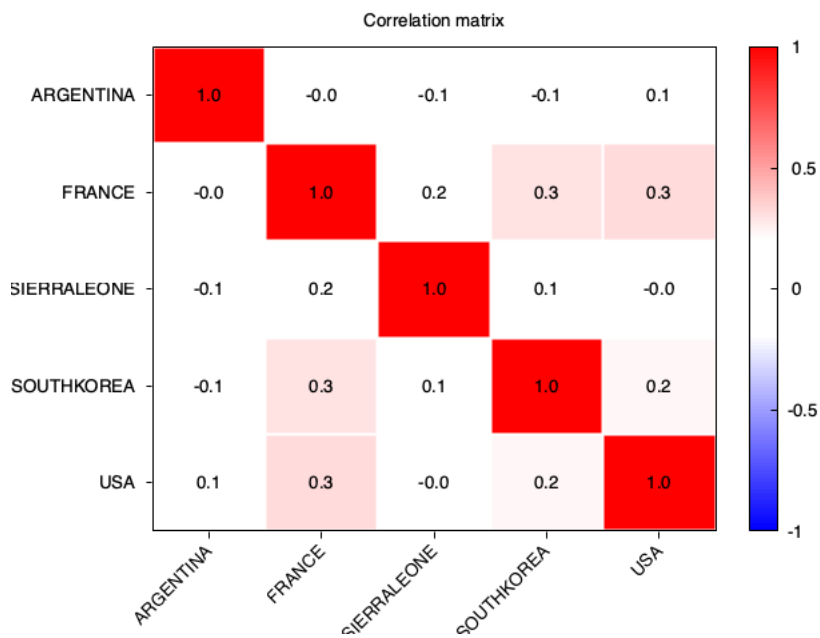
Both covariance and the correlation coefficient measure the *linear* dependence between the variables.

Notes: A correlation coefficient of 0 does not imply independence. Both covariance and correlation coefficient have the same sign.

Gretl:

View → Correlation Matrix → Highlight all variables of interest → Green Arrow → Ok → Heatmap (located in upper right hand corner of generated correlation matrix window)

```
Correlation Coefficients, using the observations 1965 - 2007
Two-tailed critical values for n = 43: 5% 0.3008, 1% 0.3887

       ARGENTINA          FRANCE     SIERRALEONE      SOUTHKOREA
          1.0000         -0.0128         -0.1015         -0.1061 ARGENTINA
                          1.0000          0.1940          0.2866 FRANCE
                                          1.0000          0.0579 SIERRALEONE
                                                          1.0000 SOUTHKOREA

             USA
          0.0614 ARGENTINA
          0.3125 FRANCE
         -0.0135 SIERRALEONE
          0.2229 SOUTHKOREA
          1.0000 USA
```

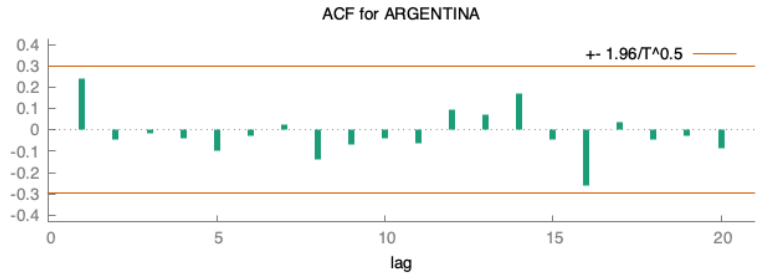


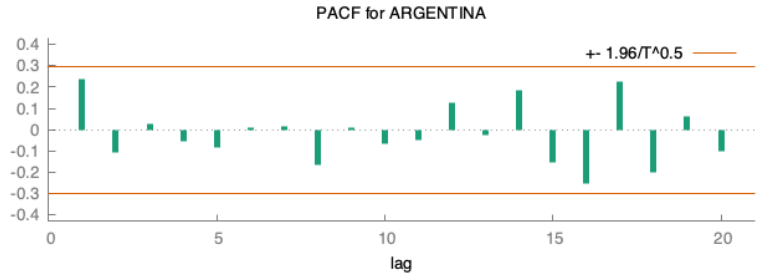
III. Correlogram

$$\rho_j = \frac{\gamma_j}{\gamma_o} \quad \text{where } \gamma_j = cov(y_t, y_{t-j)}), \quad \gamma_o = V(y_y)$$

Gretl

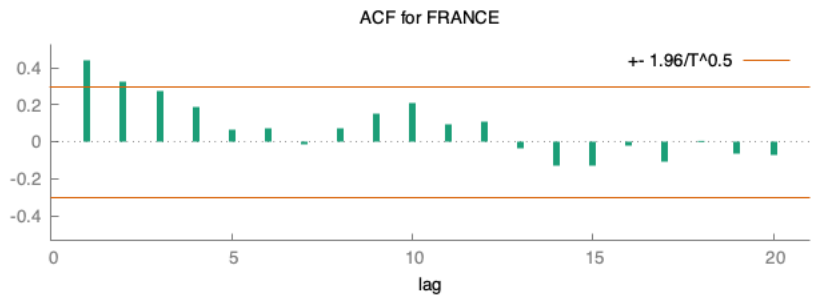
Right click variable name of interest → Correlogram → Set maximum lag equal to 20 → Ok

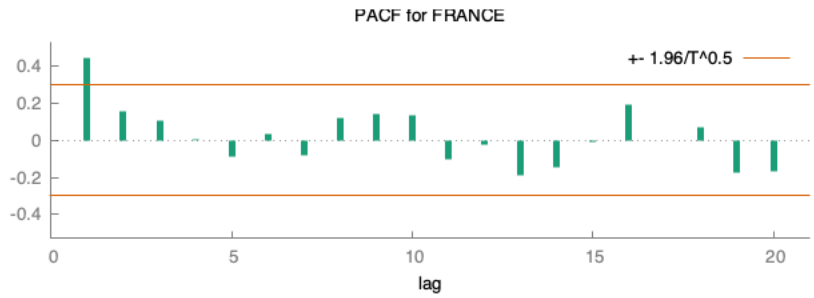




Autocorrelation function for ARGENTINA
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5

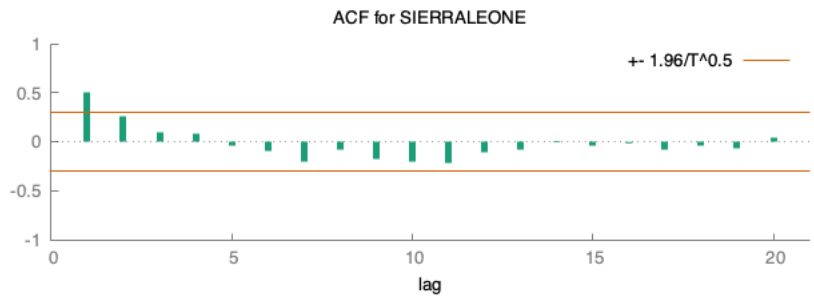
LAG	ACF		PACF	Q-stat.	[p-value]
1	0.2386		0.2386	2.6221	[0.105]
2	-0.0449		-0.1080	2.7174	[0.257]
3	-0.0143		0.0251	2.7273	[0.436]
4	-0.0413		-0.0522	2.8119	[0.590]
5	-0.0998		-0.0820	3.3187	[0.651]
6	-0.0299		0.0111	3.3653	[0.762]
7	0.0241		0.0159	3.3965	[0.846]
8	-0.1407		-0.1664	4.4905	[0.810]
9	-0.0668		0.0107	4.7443	[0.856]
10	-0.0410		-0.0649	4.8430	[0.901]
11	-0.0643		-0.0507	5.0932	[0.927]
12	0.0962		0.1290	5.6709	[0.932]
13	0.0721		-0.0271	6.0064	[0.946]
14	0.1722		0.1834	7.9846	[0.890]
15	-0.0465		-0.1518	8.1342	[0.918]
16	-0.2611	*	-0.2516	13.0192	[0.671]
17	0.0377		0.2264	13.1251	[0.728]
18	-0.0429		-0.1994	13.2673	[0.775]
19	-0.0292		0.0628	13.3359	[0.821]
20	-0.0863		-0.0991	13.9628	[0.832]

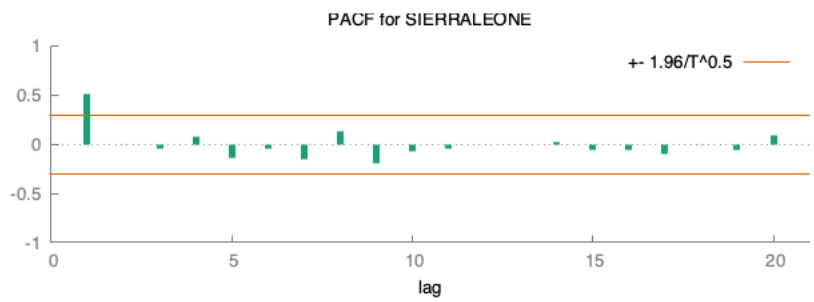




Autocorrelation function for FRANCE
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5

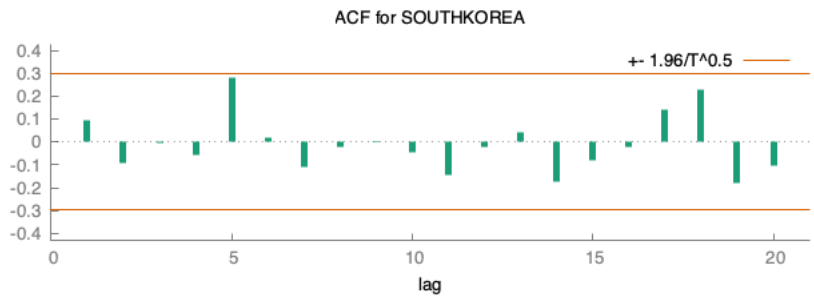
LAG	ACF		PACF		Q-stat.	[p-value]
1	0.4429	***	0.4429	***	9.0360	[0.003]
2	0.3236	**	0.1585		13.9772	[0.001]
3	0.2719	*	0.1035		17.5537	[0.001]
4	0.1857		0.0053		19.2638	[0.001]
5	0.0662		-0.0871		19.4870	[0.002]
6	0.0758		0.0323		19.7878	[0.003]
7	-0.0138		-0.0790		19.7980	[0.006]
8	0.0724		0.1216		20.0879	[0.010]
9	0.1511		0.1415		21.3872	[0.011]
10	0.2113		0.1365		24.0057	[0.008]
11	0.0931		-0.1039		24.5297	[0.011]
12	0.1110		-0.0205		25.2984	[0.013]
13	-0.0357		-0.1876		25.3806	[0.021]
14	-0.1253		-0.1477		26.4284	[0.023]
15	-0.1308		-0.0072		27.6108	[0.024]
16	-0.0183		0.1911		27.6348	[0.035]
17	-0.1029		0.0003		28.4235	[0.040]
18	0.0062		0.0730		28.4265	[0.056]
19	-0.0599		-0.1738		28.7159	[0.071]
20	-0.0694		-0.1664		29.1214	[0.085]

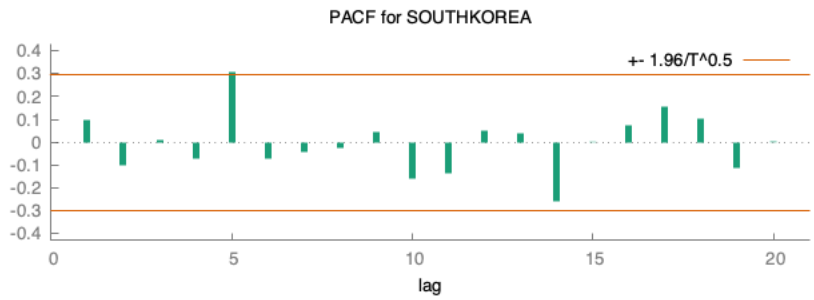




Autocorrelation function for SIERRALEONE
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5

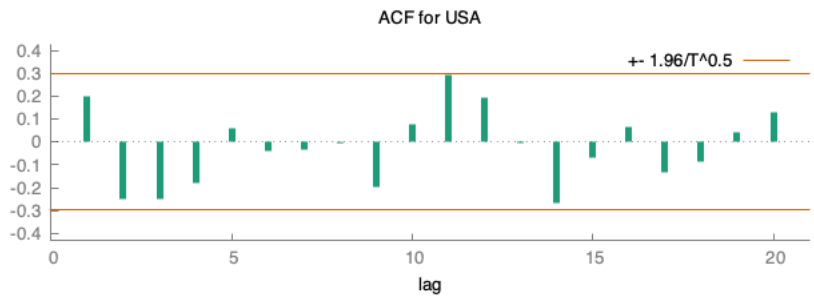
LAG	ACF		PACF		Q-stat.	[p-value]
1	0.5060	***	0.5060	***	11.7964	[0.001]
2	0.2579	*	0.0025		14.9354	[0.001]
3	0.0995		-0.0429		15.4145	[0.001]
4	0.0880		0.0716		15.7984	[0.003]
5	-0.0437		-0.1441		15.8954	[0.007]
6	-0.0941		-0.0405		16.3582	[0.012]
7	-0.2060		-0.1555		18.6393	[0.009]
8	-0.0807		0.1273		18.9997	[0.015]
9	-0.1777		-0.1978		20.7961	[0.014]
10	-0.1978		-0.0694		23.0894	[0.010]
11	-0.2114		-0.0447		25.7918	[0.007]
12	-0.1097		-0.0005		26.5425	[0.009]
13	-0.0757		0.0009		26.9120	[0.013]
14	0.0094		0.0259		26.9178	[0.020]
15	-0.0396		-0.0610		27.0263	[0.029]
16	-0.0135		-0.0631		27.0393	[0.041]
17	-0.0722		-0.0949		27.4278	[0.052]
18	-0.0346		-0.0025		27.5206	[0.070]
19	-0.0649		-0.0625		27.8606	[0.086]
20	0.0417		0.0866		28.0068	[0.109]

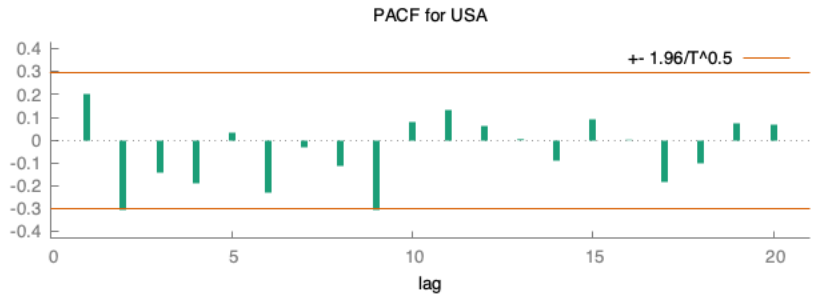




Autocorrelation function for SOUTHKOREA
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5

LAG	ACF		PACF		Q-stat.	[p-value]
1	0.0956		0.0956		0.4211	[0.516]
2	-0.0912		-0.1013		0.8141	[0.666]
3	-0.0073		0.0122		0.8167	[0.845]
4	-0.0592		-0.0703		0.9905	[0.911]
5	0.2835	*	0.3050	**	5.0843	[0.406]
6	0.0183		-0.0715		5.1018	[0.531]
7	-0.1098		-0.0449		5.7497	[0.569]
8	-0.0207		-0.0263		5.7734	[0.673]
9	0.0032		0.0469		5.7740	[0.762]
10	-0.0459		-0.1606		5.8974	[0.824]
11	-0.1472		-0.1344		7.2082	[0.782]
12	-0.0201		0.0498		7.2334	[0.842]
13	0.0442		0.0408		7.3594	[0.883]
14	-0.1755		-0.2579	*	9.4155	[0.804]
15	-0.0819		0.0026		9.8792	[0.827]
16	-0.0198		0.0745		9.9072	[0.871]
17	0.1399		0.1533		11.3648	[0.837]
18	0.2293		0.1019		15.4343	[0.632]
19	-0.1787		-0.1134		18.0087	[0.522]
20	-0.1050		0.0037		18.9370	[0.526]






```
Autocorrelation function for USA
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5
```

LAG	ACF		PACF		Q-stat.	[p-value]
1	0.2021		0.2021		1.8826	[0.170]
2	-0.2519	*	-0.3052	**	4.8774	[0.087]
3	-0.2523	*	-0.1427		7.9574	[0.047]
4	-0.1764		-0.1871		9.5018	[0.050]
5	0.0622		0.0345		9.6985	[0.084]
6	-0.0424		-0.2300		9.7924	[0.134]
7	-0.0336		-0.0289		9.8530	[0.197]
8	-0.0052		-0.1113		9.8544	[0.275]
9	-0.1990		-0.3029	**	12.1072	[0.207]
10	0.0767		0.0807		12.4525	[0.256]
11	0.2949	*	0.1318		17.7127	[0.088]
12	0.1939		0.0655		20.0604	[0.066]
13	-0.0072		0.0050		20.0637	[0.094]
14	-0.2673	*	-0.0868		24.8321	[0.036]
15	-0.0663		0.0925		25.1355	[0.048]
16	0.0630		0.0028		25.4199	[0.063]
17	-0.1304		-0.1805		26.6852	[0.063]
18	-0.0869		-0.1035		27.2697	[0.074]
19	0.0436		0.0761		27.4232	[0.095]
20	0.1274		0.0693		28.7885	[0.092]

IV. Estimating Equations

Gretl

Model → Univariate Time Series → ARIMA → Select Dependent Variable → Blue Arrow → Select ARIMA (p,d,q) order by changing "Orders" values → Ok

ARMA(0,0)

```
Model 6: ARMA, using observations 1965-2007 (T = 43)
Estimated using least squares (= MLE)
Dependent variable: SIERRALEONE

             coefficient   std. error      z      p-value
  -------------------------------------------------------
  const       -0.203721     1.14690     -0.1776   0.8590

Mean dependent var  -0.203721   S.D. dependent var   7.520704
Mean of innovations  0.000000   S.D. of innovations  7.520704
R-squared            0.351339   Adjusted R-squared   0.366424
Log-likelihood      -147.2678   Akaike criterion     298.5356
Schwarz criterion    302.0580   Hannan-Quinn         299.8346
```

ARMA(1,0)

```
Function evaluations: 22
Evaluations of gradient: 10

Model 8: ARMA, using observations 1965-2007 (T = 43)
Estimated using AS 197 (exact ML)
Dependent variable: SIERRALEONE
Standard errors based on Hessian

             coefficient   std. error      z       p-value
  --------------------------------------------------------
  const      0.00224371     1.92968     0.001163   0.9991
  phi_1      0.506608       0.130039    3.896      9.79e-05 ***

Mean dependent var  -0.203721   S.D. dependent var   7.520704
Mean of innovations -0.023495   S.D. of innovations  6.384024
R-squared            0.262420   Adjusted R-squared   0.262420
Log-likelihood      -140.8760   Akaike criterion     287.7520
Schwarz criterion    293.0356   Hannan-Quinn         289.7004

                        Real  Imaginary    Modulus  Frequency
  ------------------------------------------------------------
  AR
    Root  1           1.9739     0.0000     1.9739     0.0000
  ------------------------------------------------------------
```

ARMA(0,1)

```
Function evaluations: 23
Evaluations of gradient: 11

Model 9: ARMA, using observations 1965-2007 (T = 43)
Estimated using AS 197 (exact ML)
Dependent variable: SIERRALEONE
Standard errors based on Hessian

             coefficient   std. error      z      p-value
  ---------------------------------------------------------
  const       -0.170804     1.40550     -0.1215   0.9033
  theta_1      0.399525     0.109986     3.632    0.0003  ***

Mean dependent var  -0.203721   S.D. dependent var   7.520704
Mean of innovations  0.010365   S.D. of innovations  6.628804
R-squared            0.217332   Adjusted R-squared   0.217332
Log-likelihood      -142.4326   Akaike criterion     290.8651
Schwarz criterion    296.1487   Hannan-Quinn         292.8135

                        Real  Imaginary    Modulus  Frequency
  -----------------------------------------------------------
  MA
    Root  1          -2.5030     0.0000     2.5030     0.5000
  -----------------------------------------------------------
```

ARMA(1,1)

```
Function evaluations: 16
Evaluations of gradient: 8

Model 10: ARMA, using observations 1965-2007 (T = 43)
Estimated using AS 197 (exact ML)
Dependent variable: SIERRALEONE
Standard errors based on Hessian

             coefficient   std. error       z       p-value
  ---------------------------------------------------------
  const       0.00744663    1.94540      0.003828   0.9969
  phi_1       0.516492      0.240147     2.151      0.0315  **
  theta_1    -0.0133174     0.274905    -0.04844    0.9614

Mean dependent var  -0.203721   S.D. dependent var   7.520704
Mean of innovations -0.024148   S.D. of innovations  6.383844
R-squared            0.262464   Adjusted R-squared   0.244475
Log-likelihood      -140.8748   Akaike criterion     289.7496
Schwarz criterion    296.7944   Hannan-Quinn         292.3475

                        Real  Imaginary    Modulus  Frequency
  -----------------------------------------------------------
  AR
    Root  1           1.9361     0.0000     1.9361     0.0000
  MA
    Root  1          75.0896     0.0000    75.0896     0.0000
  -----------------------------------------------------------
```

ARMA(1,2)

```
Function evaluations: 19
Evaluations of gradient: 10

Model 11: ARMA, using observations 1965-2007 (T = 43)
Estimated using AS 197 (exact ML)
Dependent variable: SIERRALEONE
Standard errors based on Hessian

             coefficient   std. error      z       p-value
  ---------------------------------------------------------
  const      -0.0188877     1.73495     -0.01089   0.9913
  phi_1       0.0502834     0.749193     0.06712   0.9465
  theta_1     0.469041      0.740396     0.6335    0.5264
  theta_2     0.249011      0.344356     0.7231    0.4696

Mean dependent var  -0.203721   S.D. dependent var   7.520704
Mean of innovations -0.041505   S.D. of innovations  6.364890
R-squared            0.267040   Adjusted R-squared   0.230391
Log-likelihood      -140.7638   Akaike criterion     291.5277
Schwarz criterion    300.3337   Hannan-Quinn         294.7750

                        Real  Imaginary    Modulus  Frequency
  ------------------------------------------------------------
  AR
    Root  1           19.8873     0.0000    19.8873     0.0000
  MA
    Root  1           -0.9418    -1.7689     2.0040    -0.3279
    Root  2           -0.9418     1.7689     2.0040     0.3279
  ------------------------------------------------------------
```

Tables of Coefficients

Where Φ is the coefficient of AR and Θ_j is the coefficient of MA(j)

Country	Φ	Θ_1	Θ_2	Θ_3
Argentina				
ARMA(0,0)	-	-	-	-
ARMA(1,0)	0.2492	-	-	-
ARMA(0,1)	-	0.3004	-	-
ARMA(1,1)	-0.1604	0.4478	-	-
France				
ARMA(0,0)	-	-	-	-
ARMA(1,0)	0.4477	-	-	-
ARMA(0,1)	-	0.3401	-	-
ARMA(1,1)	0.7882	-0.4423	-	-

ARMA(1,3)	0.651	-0.3037	0.0333	0.0961
Sierra Leone				
ARMA(0,0)	-	-	-	-
ARMA(1,0)	0.5066	-	-	-
ARMA(0,1)	-	0.3995	-	-
ARMA(1,1)	0.5165	-0.0133	-	-
ARMA(1,2)	0.0502	0.469	0.249	-
South Korea				
ARMA(0,0)	-	-	-	-
ARMA(1,0)	0.0954	-	-	-
ARMA(0,1)	-	0.1199	-	-
ARMA(1,1)	-0.6879	0.8767	-	-
USA				
ARMA(0,0)	-	-	-	-
ARMA(1,0)	0.211	-	-	-
ARMA(0,1)	-	0.3459	-	-
ARMA(1,1)	-0.4005	0.7049	-	-

Diagnostics Table

Where adj. R^2 is the adjusted R^2, $\hat{\sigma}$ is the S.E. of Regression, A.I.C. is the Akaike Info Criterion, and S.C. is the Schwarz Criterion

Country	Adj. R^2	$\hat{\sigma}$	A.I.C	S.C.
Argentina				
ARMA(0,0)	0.0237	4.7613	259.222	262.744
ARMA(1,0)	0.061	4.5601	258.584	263.867
ARMA(0,1)	0.0738	4.5294	258.033	263.317
ARMA(1,1)	0.0533	4.5239	259.937	266.981
France				
ARMA(0,0)	0	1.7471	173.004	176.526
ARMA(1,0)	0.2031	1.5416	165.475	170.758
ARMA(0,1)	0.1537	1.5947	168.287	173.57
ARMA(1,1)	0.2191	1.5085	165.717	172.762

ARMA(1,3)	0.1811	1.5061	169.583	180.151
Sierra Leone				
ARMA(0,0)	0.3664	7.5207	298.536	302.058
ARMA(1,0)	0.2624	6.384	287.752	293.036
ARMA(0,1)	0.2173	6.6288	290.865	296.149
ARMA(1,1)	0.2444	6.3838	289.75	296.794
ARMA(1,2)	0.2304	6.3649	291.528	300.333
South Korea				
ARMA(0,0)	0.0235	4.5189	254.729	258.251
ARMA(1,0)	0.0093	4.4452	256.335	261.619
ARMA(0,1)	0.0118	4.4399	256.237	261.52
ARMA(1,1)	0.0428	4.3175	256.075	263.12
USA				
ARMA(0,0)	0	2.0753	187.805	191.328
ARMA(1,0)	0.0437	2.0057	187.93	193.214
ARMA(0,1)	0.0807	1.969	186.414	191.698
ARMA(1,1)	0.0808	1.943	187.351	194.396

** Of the Sierra Leone models, we select the ARMA (1,0) based on the above diagnostics. The rest of this packet is based on this model specification.*

V. Forecasting

In-Sample Forecasting

Gretl

Model → Univariate Time Series → ARIMA → Select Dependent Variable → Blue Arrow → Select ARIMA (p,d,q) order by changing "Orders" values → Ok → Analysis → Display actual, fitted, residual

To show residual graph

(Starting after step "Ok" from above) → Graphs → Residual Plot → Against Time

To show fitted vs. actual graph

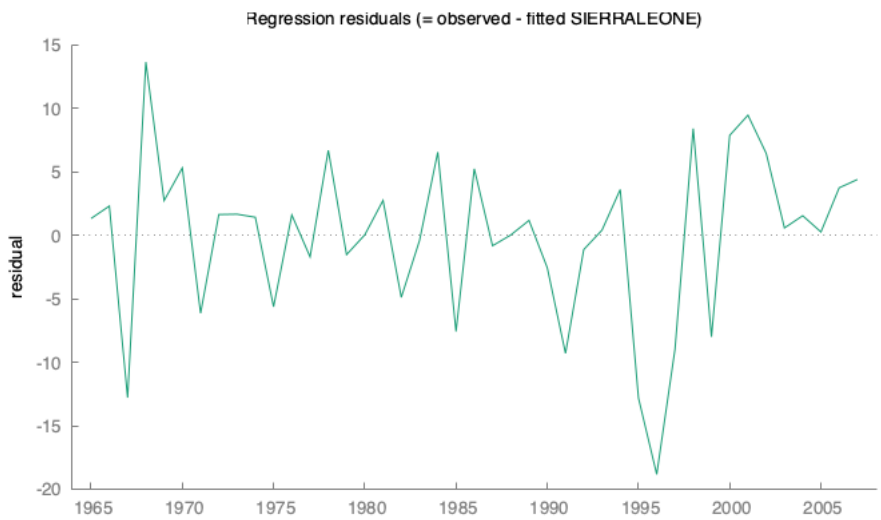
(Starting after step "Ok" from above) → Graphs → Fitted, Actual Plot → Against Time

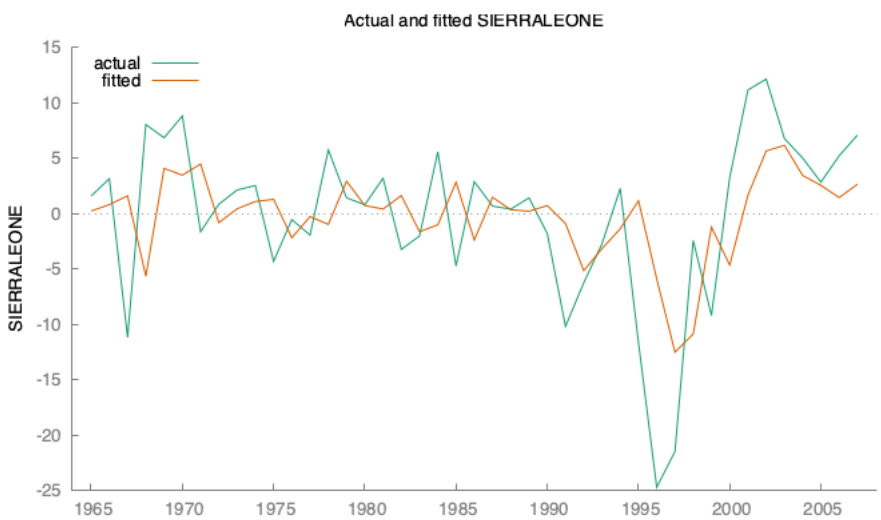
Model estimation range: 1965 - 2007
Standard error of the regression = 6.38402

	SIERRALEONE	fitted	residual	
1965	1.57	0.22	1.35	
1966	3.13	0.80	2.33	
1967	-11.17	1.59	-12.76	
1968	8.01	-5.66	13.67	
1969	6.82	4.06	2.76	
1970	8.79	3.46	5.33	
1971	-1.65	4.45	-6.10	
1972	0.83	-0.83	1.66	
1973	2.11	0.42	1.69	
1974	2.52	1.07	1.45	
1975	-4.32	1.28	-5.60	
1976	-0.56	-2.19	1.63	
1977	-1.95	-0.28	-1.67	
1978	5.73	-0.99	6.72	
1979	1.41	2.90	-1.49	
1980	0.78	0.72	0.06	
1981	3.17	0.40	2.77	
1982	-3.25	1.61	-4.86	
1983	-2.03	-1.65	-0.38	
1984	5.55	-1.03	6.58	
1985	-4.73	2.81	-7.54	
1986	2.87	-2.40	5.27	
1987	0.66	1.46	-0.80	
1988	0.39	0.34	0.05	
1989	1.40	0.20	1.20	
1990	-1.81	0.71	-2.52	
1991	-10.18	-0.92	-9.26	
1992	-6.25	-5.16	-1.09	
1993	-2.73	-3.17	0.44	
1994	2.25	-1.38	3.63	
1995	-11.64	1.14	-12.78	
1996	-24.69	-5.90	-18.79	*
1997	-21.46	-12.51	-8.95	
1998	-2.44	-10.87	8.43	
1999	-9.20	-1.24	-7.96	
2000	3.24	-4.66	7.90	
2001	11.13	1.64	9.49	
2002	12.12	5.64	6.48	
2003	6.75	6.14	0.61	
2004	4.99	3.42	1.57	
2005	2.82	2.53	0.29	
2006	5.20	1.43	3.77	
2007	7.06	2.64	4.42	

Note: * denotes a residual in excess of 2.5 standard errors
Forecast evaluation statistics using 43 observations

Mean Error	-0.023495
Root Mean Squared Error	6.384
Mean Absolute Error	4.7473
Mean Percentage Error	65.717
Mean Absolute Percentage Error	106.58
Theil's U2	0.97468



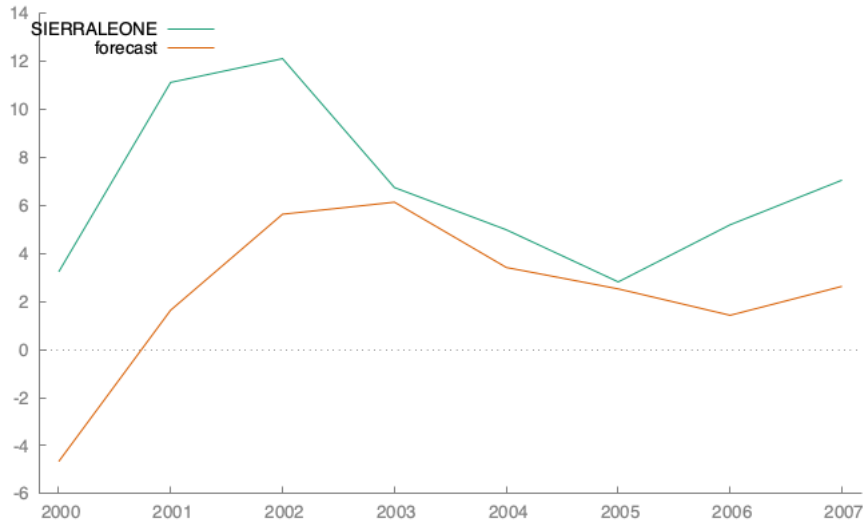


Out of Sample Forecasting

Gretl

> *(Starting after step "Ok" from in sample forecasting) → Analysis → Forecasts → (if prompted to add observations, enter number of forecast periods of interest) → Select forecast range (2000-2007) → Static Forecast → Select 0 pre-forecast observations to graph → Ok*

Note: selecting forecast range (dates) that exist in the original data file (up through 2007 here) will present the following results with forecasting performance metrics.



	SIERRALEONE	prediction
2000	3.24	-4.66
2001	11.13	1.64
2002	12.12	5.64
2003	6.75	6.14
2004	4.99	3.42
2005	2.82	2.53
2006	5.20	1.43
2007	7.06	2.64

Forecast evaluation statistics using 8 observations

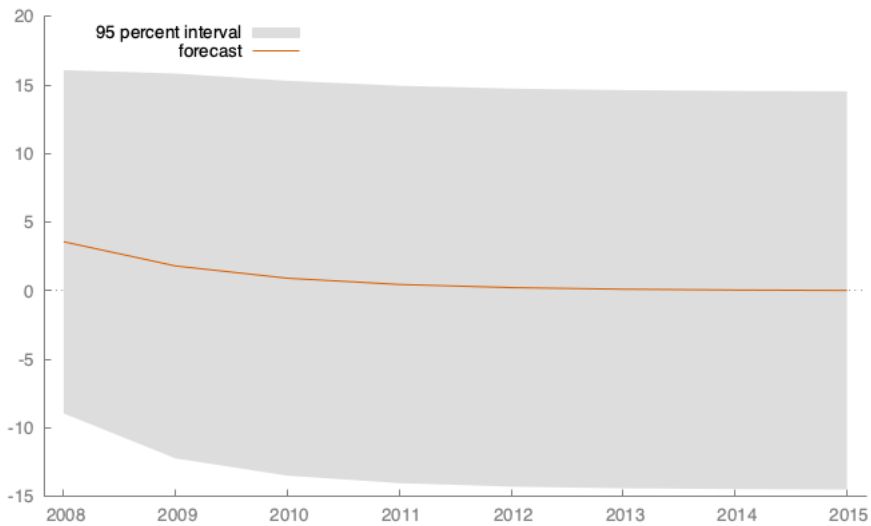
Mean Error	4.3164
Root Mean Squared Error	5.375
Mean Absolute Error	4.3164
Mean Percentage Error	71.061
Mean Absolute Percentage Error	71.061
Theil's U2	1.2602
Bias proportion, UM	0.64491
Regression proportion, UR	0.084847
Disturbance proportion, UD	0.27025

Year	Actual	Forecast	Forecast Error
2000	3.24	-4.66	7.9
2001	11.13	1.64	9.49
2002	12.12	5.64	6.48
2003	6.75	6.14	.61
2004	4.99	3.42	1.57
2005	2.82	2.53	0.29
2006	5.2	1.43	3.77
2007	7.06	2.64	4.42

Note: the following demonstrates output that is generated by selecting a forecast range that extends past the most recent data point available in the file (2007 here).

Gretl

(Starting after step "Ok" from in sample forecasting) → Analysis → Forecasts → (if prompted to add observations, enter number of forecast periods of interest, here 8) → Select forecast range (2008-2015) → Dynamic Forecast → Select 0 pre-forecast observations to graph → Plot using shaded area → Ok



For 95% confidence intervals, z(0.025) = 1.96

	SIERRALEONE	prediction	std. error	95% interval
2008		3.57776	6.38402	−8.93470 - 16.0902
2009		1.81363	7.15652	−12.2129 - 15.8402
2010		0.919906	7.34169	−13.4695 - 15.3093
2011		0.467139	7.38846	−14.0140 - 14.9483
2012		0.237763	7.40042	−14.2668 - 14.7423
2013		0.121560	7.40349	−14.3890 - 14.6321
2014		0.0626902	7.40427	−14.4494 - 14.5748
2015		0.0328664	7.40447	−14.4796 - 14.5454

Multivariate Time Series Models

Gretl

Model → Univariate Time Series → ARIMA → Select Dependent Variable → Blue Arrow → Select any independent variables (here, USA) → Green Arrow → Select ARIMA (p,d,q) order by changing "Orders" values (here, 1,0,0) → Ok

```
Function evaluations: 17
Evaluations of gradient: 7

Model 2: ARMAX, using observations 1965-2007 (T = 43)
Estimated using AS 197 (exact ML)
Dependent variable: SIERRALEONE
Standard errors based on Hessian

             coefficient   std. error      z      p-value
  ---------------------------------------------------------
  const       -1.15630      2.28552     -0.5059   0.6129
  phi_1        0.546318     0.129136     4.231    2.33e-05 ***
  USA          0.544335     0.464405     1.172    0.2412

Mean dependent var  -0.203721   S.D. dependent var   7.520704
Mean of innovations  0.001897   S.D. of innovations  6.282557
R-squared            0.286060   Adjusted R-squared   0.268647
Log-likelihood      -140.2160   Akaike criterion     288.4320
Schwarz criterion    295.4768   Hannan-Quinn         291.0299

                        Real  Imaginary    Modulus  Frequency
  ------------------------------------------------------------
  AR
    Root  1           1.8304     0.0000     1.8304     0.0000
  ------------------------------------------------------------
```

Note: To add lags of an independent variable, follow the steps below.

Gretl

(Beginning with step "Green Arrow" from above) → Click "lags"→ Specify the lags to model (here, USA from 1 to 1) → Select ARIMA (p,d,q) order by changing "Orders" values (here, 1,0,2) → Ok

```
Function evaluations: 15
Evaluations of gradient: 8

Model 3: ARMAX, using observations 1966-2007 (T = 42)
Estimated using AS 197 (exact ML)
Dependent variable: SIERRALEONE
Standard errors based on Hessian

             coefficient   std. error       z       p-value
  ----------------------------------------------------------
  const       0.0148188     2.24739      0.006594   0.9947
  phi_1       0.506387      0.131892     3.839      0.0001  ***
  USA_1      -0.00533692    0.474815    -0.01124    0.9910

Mean dependent var  -0.245952   S.D. dependent var   7.606705
Mean of innovations -0.048202   S.D. of innovations  6.459590
R-squared            0.261430   Adjusted R-squared   0.242965
Log-likelihood      -138.0973   Akaike criterion     284.1947
Schwarz criterion    291.1453   Hannan-Quinn         286.7423

                        Real  Imaginary    Modulus  Frequency
  ------------------------------------------------------------
  AR
    Root  1           1.9748     0.0000     1.9748     0.0000
  ------------------------------------------------------------
```

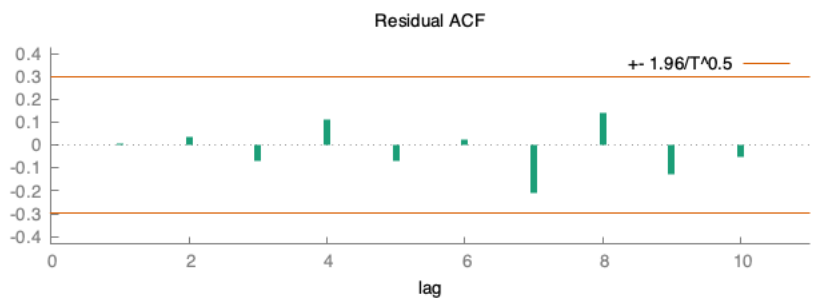
VI. Diagnostic Checking

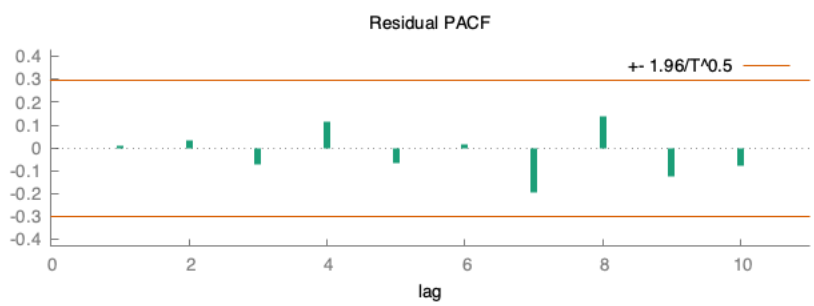
Residual Diagnostics

i. Correlogram of Residuals

Gretl

(Beginning with Sierra Leone ARMA (1,0) estimated equation window) → Graphs → Residual correlogram → Select number of lags → Ok





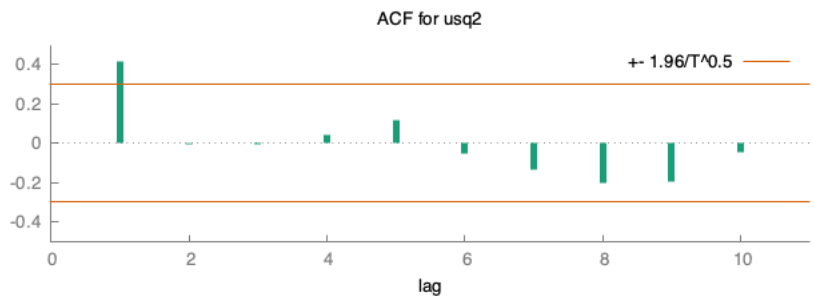

```
Residual autocorrelation function
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5
```

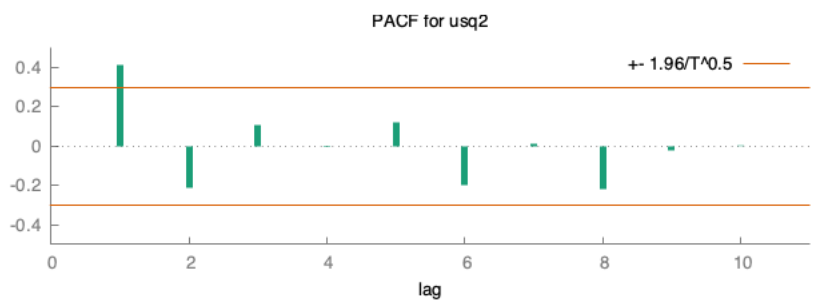
LAG	ACF	PACF	Q-stat.	[p-value]
1	0.0073	0.0073		
2	0.0343	0.0343	0.0581	[0.810]
3	-0.0698	-0.0704	0.2941	[0.863]
4	0.1150	0.1157	0.9501	[0.813]
5	-0.0690	-0.0686	1.1924	[0.879]
6	0.0273	0.0182	1.2315	[0.942]
7	-0.2097	-0.1954	3.5953	[0.731]
8	0.1442	0.1401	4.7454	[0.691]
9	-0.1267	-0.1222	5.6584	[0.685]
10	-0.0488	-0.0766	5.7980	[0.760]

ii. Correlogram of Residuals Squared

Gretl

(Beginning with estimated equation window) → Save → Squared residuals → Select variable save name (here, usq2) → Ok → Return to home screen with all variables listed → Right click variable saved in step 4 (here, usq2) → Correlogram → Select number of lags → Ok





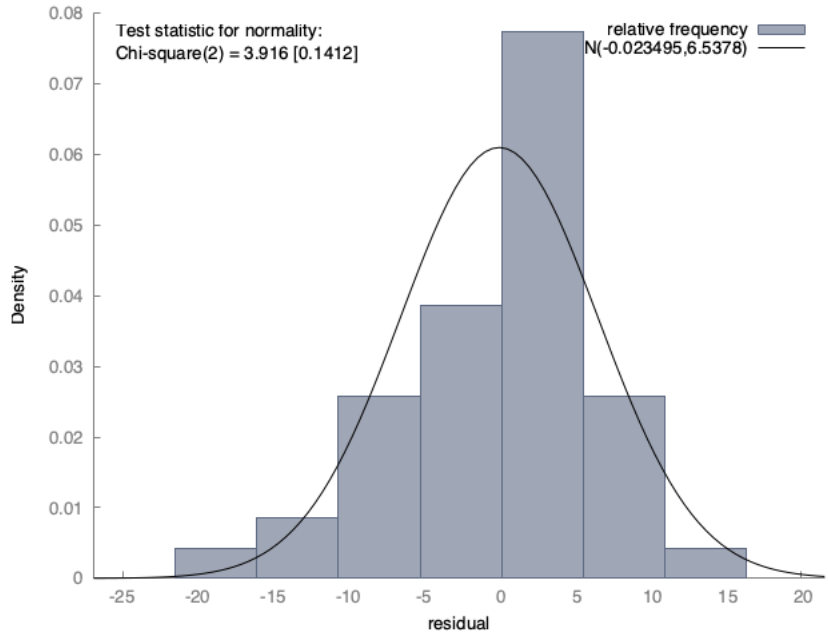

```
Autocorrelation function for usq2
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5
```

LAG	ACF		PACF		Q-stat.	[p-value]
1	0.4154	***	0.4154	***	7.9516	[0.005]
2	-0.0055		-0.2153		7.9530	[0.019]
3	-0.0075		0.1074		7.9557	[0.047]
4	0.0393		-0.0036		8.0325	[0.090]
5	0.1164		0.1226		8.7220	[0.121]
6	-0.0547		-0.2022		8.8784	[0.181]
7	-0.1342		0.0112		9.8468	[0.197]
8	-0.2019		-0.2210		12.1002	[0.147]
9	-0.1932		-0.0239		14.2252	[0.115]
10	-0.0457		0.0030		14.3478	[0.158]

iii. Histogram of Residuals

Gretl

(Beginning with estimated equation window) → Tests → Normality of Residuals




```
Frequency distribution for residual, obs 1-43
number of bins = 7, mean = -0.0234947, sd = 6.53783

       interval          midpt   frequency    rel.     cum.

            < -16.089   -18.794        1      2.33%    2.33%
   -16.089 - -10.679   -13.384        2      4.65%    6.98% *
   -10.679 - -5.2684   -7.9736        6     13.95%   20.93% *****
   -5.2684 -  0.14192  -2.5632        9     20.93%   41.86% *******
   0.14192 -  5.5522    2.8471       18     41.86%   83.72% ***************
    5.5522 -  10.963    8.2574        6     13.95%   97.67% *****
          >=  10.963    13.668        1      2.33%  100.00%

Test for null hypothesis of normal distribution:
Chi-square(2) = 3.916 with p-value 0.14115
```

iv. Coefficient Diagnostics

Gretl

NOTE: This example uses the Sierra Leone ARMA (1,2) to test statistical significance of ma(2) term.

(Beginning with estimated equation window) → Tests → Linear Restrictions → Enter restrictions (click help for specification assistance), (here, enter b[theta_2] = 0 to test whether the ma(2) term is statistically significant) → Ok

```
Restriction:
 b[theta_2] = 0

Test statistic: chi^2(1) = 32.4885, with p-value = 1.19899e-08
```

VII. Non-Stationary Models

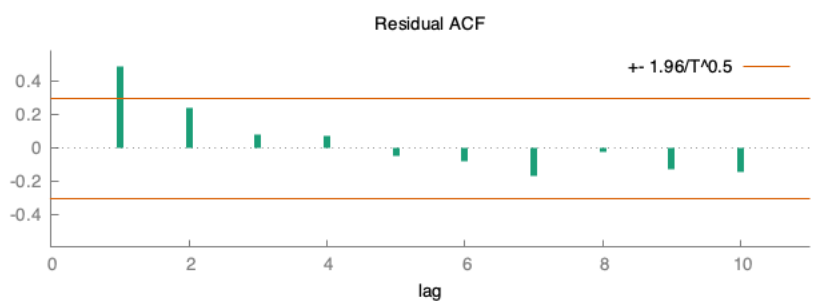
i. Detrending

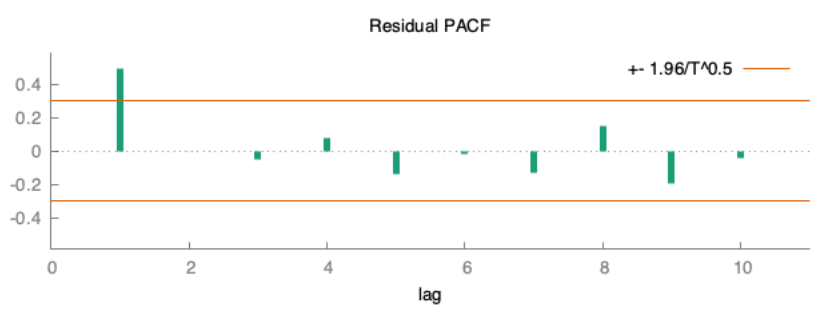
$$y_t = a_0 + a_1 t + a_2 t^2 + z_t$$

$$\hat{z}_t = y_t - \hat{a}_0 - \hat{a}_1 t - \hat{a}_2 t^2$$

Gretl

*Add → Time trend → Right click new variable called "time" → Define new variable → Enter: time2 = time * time → Ok → Model → Ordinary Least Squares → Click Sierra Leone → Blue arrow (dependent variable) → Select time and time2 → Green arrow (independent variables/regressors) → Ok → Graphs → Residual Correlogram → Ok*





```
Residual autocorrelation function
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5

  LAG      ACF          PACF         Q-stat. [p-value]

    1   0.4940  ***   0.4940 ***     11.2421  [0.001]
    2   0.2434       -0.0008         14.0374  [0.001]
    3   0.0788       -0.0543         14.3381  [0.002]
    4   0.0769        0.0776         14.6318  [0.006]
    5  -0.0464       -0.1366         14.7412  [0.012]
    6  -0.0748       -0.0176         15.0337  [0.020]
    7  -0.1696       -0.1303         16.5800  [0.020]
    8  -0.0256        0.1503         16.6163  [0.034]
    9  -0.1294       -0.1918         17.5693  [0.041]
   10  -0.1439       -0.0461         18.7840  [0.043]
```

ii. Differencing

$$y_t = \propto + \beta x_t + \varepsilon_t$$

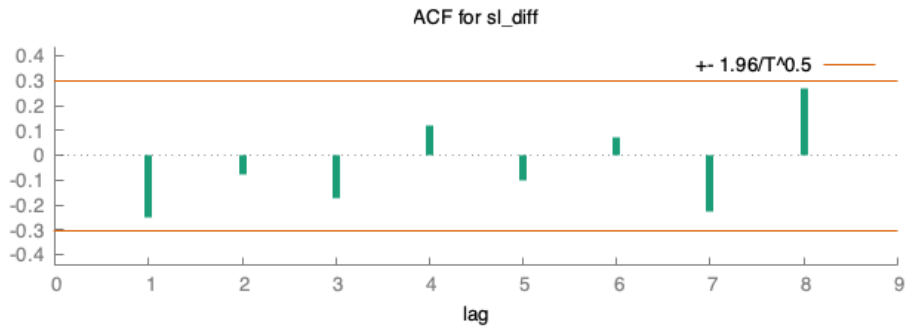
$$y_{t-1} = \propto + \beta x_{t-1} + \varepsilon_{t-1}$$

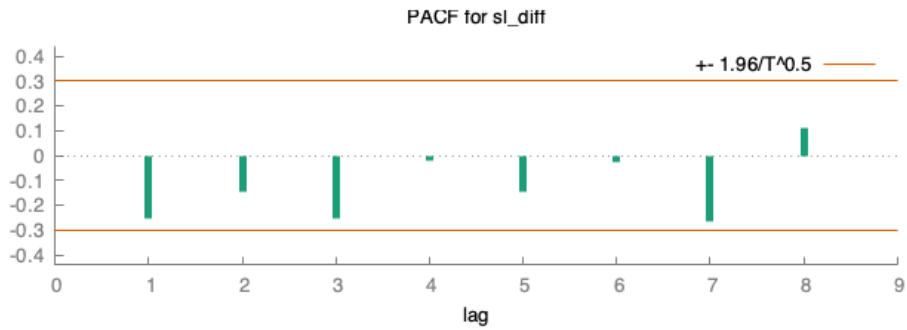
Differency Series = $y_t - y_{t-1}$

Gretl

Select Sierra Leone → Add → Lags of Selected Variables → Ok → Add → Define new variable → Enter: sl_diff = SIERRALEONE – SIERRALEONE_1 → Right click sl_diff → Correlogram → Ok

```
Autocorrelation function for sl_diff
***, **, * indicate significance at the 1%, 5%, 10% levels
using standard error 1/T^0.5

  LAG      ACF          PACF         Q-stat. [p-value]

    1  -0.2505       -0.2505          2.8275  [0.093]
    2  -0.0752       -0.1472          3.0891  [0.213]
    3  -0.1739       -0.2534          4.5217  [0.210]
    4   0.1198       -0.0194          5.2201  [0.265]
    5  -0.1030       -0.1480          5.7500  [0.331]
    6   0.0743       -0.0257          6.0332  [0.419]
    7  -0.2231       -0.2632 *        8.6620  [0.278]
    8   0.2695  *     0.1119         12.6102  [0.126]
```





VIII. Unit Root Test

i. Augmented Dickey-Fuller Test

Gretl

Select Sierra Leone → Variable → Unit Root Tests → Augmented Dickey-Fuller Test

```
Augmented Dickey-Fuller test for SIERRALEONE
testing down from 9 lags, criterion AIC
sample size 42
unit-root null hypothesis: a = 1

  test with constant
  including 0 lags of (1-L)SIERRALEONE
  model: (1-L)y = b0 + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.482349
  test statistic: tau_c(1) = -3.51342
  asymptotic p-value 0.007677
  1st-order autocorrelation coeff. for e: -0.006

  with constant and trend
  including 0 lags of (1-L)SIERRALEONE
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.482163
  test statistic: tau_ct(1) = -3.44955
  asymptotic p-value 0.04502
  1st-order autocorrelation coeff. for e: -0.006
```

ii. Dickey-Fuller Test (Dickey-Fuller GLS)

Gretl

Select Sierra Leone → Variable → Unit Root Tests → ADF-GLS Test

```
Augmented Dickey-Fuller (GLS) test for SIERRALEONE
testing down from 9 lags, criterion modified AIC, Perron-Qu
sample size 42
unit-root null hypothesis: a = 1

  test with constant
  including 0 lags of (1-L)SIERRALEONE
  model: (1-L)y = b0 + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.470159
  test statistic: tau = -3.51246
  approximate p-value 0.001
  1st-order autocorrelation coeff. for e: -0.010
```

IX. Structural Break Test

i. Installation of StrucBreak package

Gretl

File → Function Packages → On server → Find 'StrucBreak' → Right click → Install

ii. Bai – Perron Structural Break Test

Note: StrucBreak package must be installed before progressing to this step.

Gretl

Model → Univariate Time Series → Structural Breaks (Bai-Perron) → Select dependent variable (here, Sierra Leone) → Select constant variable(s) (here, const) → Select robust var estimate (here, HAC-robust variable estimate) → Ok

Note: Here, we are simply testing whether there are any structural breaks in the raw Sierra Leone time series. If testing against a model specification (for instance, ARMA (1,0)) add lagged Sierra Leone variable as "Z" regressor. A "Z" regressor is one whose coefficients are allowed to change to determine the break points, while "X" regressors are not allowed to vary. More information on this package can be found at: https://gretl.sourceforge.net/current_fnfiles/unzipped/StrucBreak.pdf.

```
-----------------------------------------------------------------
   OUTPUT FROM THE GLOBAL OPTIMIZATION STAGE
-----------------------------------------------------------------
Breaks         SSR      Dates

     1  1912.02498        35 (1999)
     2  1056.19105        30 (1994),   35 (1999)

-----------------------------------------------------------------
   OUTPUT FROM THE APPLICATION OF INFORMATION CRITERIA
-----------------------------------------------------------------
   Breaks     BIC     LWZ
       0   4.012   4.035
       1    3.97   4.092
       2   3.551   3.774

The number of breaks chosen by BIC is : 2
  ... chosen by Liu/Wu/Zidek (LWZ) is : 2

-------------------------------------------------------------------
   OUTPUT FROM THE TESTING PROCEDURES
-------------------------------------------------------------------
a) supF tests against a fixed number of breaks
- - - - - - - - - - - - - - - - - - - - - - - - - -
    supF(1|0)   supF(2|0)
      14.612      14.407

Critical values:
           supF(1|0)   supF(2|0)
10%          7.42        6.93
 5%          9.10        7.92
 2.5%       10.56        8.90
 1%         13.00       10.14

- - - - - - - - - - - - - - - - - - - - - - - - - -
b) Dmax tests against an unknown number of breaks
- - - - - - - - - - - - - - - - - - - - - - - - - -
UDmax test: 14.612155
Crit. values: 10%: 8.05 | 5%: 9.52 | 2.5%: 10.83 | 1%: 13.07 |

        WDmax test      (crit. val.)
10%        15.43            8.63
 5%        16.55           10.39
 2.5%      17.09           12.06
 1%        18.47           14.53

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
c) supF(l+1|l) tests using global optimizers under the null
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
supF(2|1)       11.15   1994

Critical values:   10%    5%  2.5%    1%
       supF(2|1)   9.05 10.55 12.37 14.51
```

X. Vector Autoregression

Gretl

Model → Multivariate Time Series → Vector Autoregression → Select endogenous variables (here, Argentina, France, Sierra Leone, South Korea, USA) → Green Arrow → Select Lag Order (here, 2) → Ok

Note: Granger Causality Tests are synonymous with "F-tests of zero restrictions" below. Null hypothesis of this test is that no lags of variable j (other countries) are significant in the equation for variable i (country of interest).

```
VAR system, lag order 2
OLS estimates, observations 1967-2007 (T = 41)
Log-likelihood = -501.45423
Determinant of covariance matrix = 28905.771
AIC = 27.1441
BIC = 29.4428
HQC = 27.9812
Portmanteau test: LB(10) = 227.9, df = 200 [0.0857]

Equation 1: ARGENTINA

                  coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------------
  const            2.21307       1.96854      1.124    0.2698
  ARGENTINA_1      0.300778      0.174988     1.719    0.0959  *
  ARGENTINA_2     -0.0278538     0.176823    -0.1575   0.8759
  FRANCE_1         0.195999      0.586772     0.3340   0.7407
  FRANCE_2        -0.564116      0.558317    -1.010    0.3204
  SIERRALEONE_1   -0.152181      0.123639    -1.231    0.2279
  SIERRALEONE_2    0.152182      0.138955     1.095    0.2821
  SOUTHKOREA_1     0.163674      0.201215     0.8134   0.4224
  SOUTHKOREA_2    -0.0892670     0.185608    -0.4809   0.6340
  USA_1           -0.132181      0.504332    -0.2621   0.7950
  USA_2           -0.220926      0.429463    -0.5144   0.6107

Mean dependent var   1.346341   S.D. dependent var   4.744683
Sum squared resid    700.4746   S.E. of regression   4.832096
R-squared            0.222110   Adjusted R-squared  -0.037186
F(10, 30)            0.856587   P-value(F)           0.581174
rho                  0.000645   Durbin-Watson        1.986123

F-tests of zero restrictions:

All lags of ARGENTINA        F(2, 30) =   1.5157 [0.2360]
All lags of FRANCE           F(2, 30) =  0.51753 [0.6012]
All lags of SIERRALEONE      F(2, 30) =  0.89542 [0.4191]
All lags of SOUTHKOREA       F(2, 30) =  0.41117 [0.6666]
All lags of USA              F(2, 30) =  0.19785 [0.8216]
All vars, lag 2              F(5, 30) =  0.66932 [0.6497]
```

Equation 2: FRANCE

	coefficient	std. error	t-ratio	p-value	
const	0.489213	0.608671	0.8037	0.4279	
ARGENTINA_1	-0.0349753	0.0541061	-0.6464	0.5229	
ARGENTINA_2	0.0416140	0.0546734	0.7611	0.4525	
FRANCE_1	0.246753	0.181430	1.360	0.1839	
FRANCE_2	0.417780	0.172631	2.420	0.0218	**
SIERRALEONE_1	-0.0267927	0.0382290	-0.7008	0.4888	
SIERRALEONE_2	0.0153448	0.0429648	0.3571	0.7235	
SOUTHKOREA_1	-0.0382163	0.0622154	-0.6143	0.5437	
SOUTHKOREA_2	-0.0321352	0.0573897	-0.5599	0.5797	
USA_1	0.421740	0.155939	2.705	0.0112	**
USA_2	-0.144850	0.132790	-1.091	0.2840	

Mean dependent var	2.183171	S.D. dependent var	1.747583
Sum squared resid	66.96832	S.E. of regression	1.494081
R-squared	0.451807	Adjusted R-squared	0.269076
F(10, 30)	2.472523	P-value(F)	0.026997
rho	-0.060038	Durbin-Watson	2.102299

F-tests of zero restrictions:

All lags of ARGENTINA	F(2, 30) = 0.40125	[0.6730]
All lags of FRANCE	F(2, 30) = 6.5368	[0.0044]
All lags of SIERRALEONE	F(2, 30) = 0.24570	[0.7837]
All lags of SOUTHKOREA	F(2, 30) = 0.38408	[0.6844]
All lags of USA	F(2, 30) = 3.8417	[0.0327]
All vars, lag 2	F(5, 30) = 1.5879	[0.1937]

Equation 3: SIERRALEONE

	coefficient	std. error	t-ratio	p-value	
const	-0.406878	2.93070	-0.1388	0.8905	
ARGENTINA_1	0.259761	0.260516	0.9971	0.3267	
ARGENTINA_2	-0.334356	0.263248	-1.270	0.2138	
FRANCE_1	-0.310965	0.873568	-0.3560	0.7244	
FRANCE_2	1.08762	0.831205	1.308	0.2006	
SIERRALEONE_1	0.496502	0.184070	2.697	0.0114	**
SIERRALEONE_2	0.0563327	0.206872	0.2723	0.7873	
SOUTHKOREA_1	-0.277792	0.299562	-0.9273	0.3612	
SOUTHKOREA_2	-0.141874	0.276327	-0.5134	0.6114	
USA_1	0.510401	0.750834	0.6798	0.5019	
USA_2	0.0253536	0.639371	0.03965	0.9686	

Mean dependent var	-0.328293	S.D. dependent var	7.682229
Sum squared resid	1552.554	S.E. of regression	7.193871
R-squared	0.342324	Adjusted R-squared	0.123098
F(10, 30)	1.561515	P-value(F)	0.166721
rho	-0.069469	Durbin-Watson	2.038738

F-tests of zero restrictions:

All lags of ARGENTINA	F(2, 30) = 1.0545	[0.3609]
All lags of FRANCE	F(2, 30) = 0.88473	[0.4233]
All lags of SIERRALEONE	F(2, 30) = 5.6257	[0.0084]
All lags of SOUTHKOREA	F(2, 30) = 0.61614	[0.5467]
All lags of USA	F(2, 30) = 0.24491	[0.7843]
All vars, lag 2	F(5, 30) = 0.60473	[0.6967]

Equation 4: SOUTHKOREA

	coefficient	std. error	t-ratio	p-value	
const	5.23856	1.96424	2.667	0.0122	**
ARGENTINA_1	-0.247388	0.174606	-1.417	0.1668	
ARGENTINA_2	0.00992119	0.176437	0.05623	0.9555	
FRANCE_1	0.0977927	0.585492	0.1670	0.8685	
FRANCE_2	0.185543	0.557099	0.3331	0.7414	
SIERRALEONE_1	0.129251	0.123369	1.048	0.3032	
SIERRALEONE_2	-0.0402918	0.138652	-0.2906	0.7734	
SOUTHKOREA_1	0.0731492	0.200776	0.3643	0.7182	
SOUTHKOREA_2	-0.0341549	0.185203	-0.1844	0.8549	
USA_1	0.188163	0.503232	0.3739	0.7111	
USA_2	-0.145318	0.428526	-0.3391	0.7369	

Mean dependent var	5.894146	S.D. dependent var	4.532839
Sum squared resid	697.4217	S.E. of regression	4.821555
R-squared	0.151416	Adjusted R-squared	-0.131446
F(10, 30)	0.535300	P-value(F)	0.851245
rho	0.047795	Durbin-Watson	1.874256

F-tests of zero restrictions:

All lags of ARGENTINA	F(2, 30) = 1.0501	[0.3624]
All lags of FRANCE	F(2, 30) = 0.11578	[0.8911]
All lags of SIERRALEONE	F(2, 30) = 0.59625	[0.5573]
All lags of SOUTHKOREA	F(2, 30) = 0.077378	[0.9257]
All lags of USA	F(2, 30) = 0.10786	[0.8981]
All vars, lag 2	F(5, 30) = 0.068676	[0.9964]

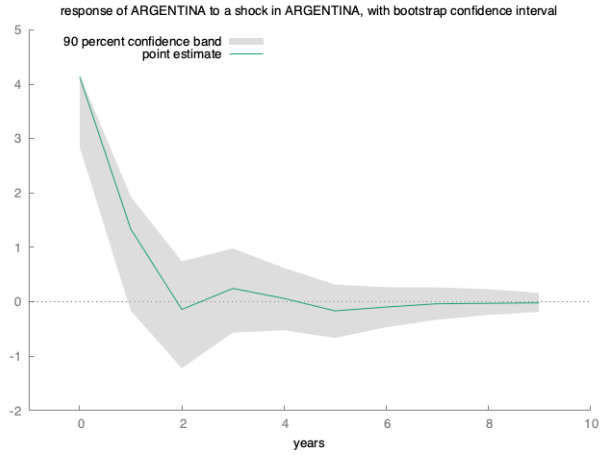
```
Equation 5: USA

                  coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------------
  const            2.04310      0.769704      2.654    0.0126  **
  ARGENTINA_1     -0.00907030   0.0684207    -0.1326   0.8954
  ARGENTINA_2      0.0192532    0.0691382     0.2785   0.7826
  FRANCE_1        -0.136444     0.229430     -0.5947   0.5565
  FRANCE_2         0.0968216    0.218304      0.4435   0.6606
  SIERRALEONE_1   -0.114114     0.0483431    -2.360    0.0249  **
  SIERRALEONE_2    0.0597645    0.0543319     1.100    0.2801
  SOUTHKOREA_1    -0.110374     0.0786755    -1.403    0.1709
  SOUTHKOREA_2     0.0743304    0.0725731     1.024    0.3139
  USA_1            0.445683     0.197195      2.260    0.0312  **
  USA_2           -0.321019     0.167921     -1.912    0.0655  *

Mean dependent var   2.004634   S.D. dependent var   2.015738
Sum squared resid    107.0909   S.E. of regression   1.889364
R-squared            0.341093   Adjusted R-squared   0.121457
F(10, 30)            1.552995   P-value(F)           0.169521
rho                  0.006086   Durbin-Watson        1.979003

F-tests of zero restrictions:

All lags of ARGENTINA        F(2, 30) = 0.040922 [0.9600]
All lags of FRANCE           F(2, 30) =  0.19772 [0.8217]
All lags of SIERRALEONE      F(2, 30) =   2.8004 [0.0767]
All lags of SOUTHKOREA       F(2, 30) =   1.3775 [0.2677]
All lags of USA              F(2, 30) =   3.7167 [0.0361]
All vars, lag 2              F(5, 30) =   1.0244 [0.4210]

For the system as a whole:

  Null hypothesis: the longest lag is 1
  Alternative hypothesis: the longest lag is 2
  Likelihood ratio test: Chi-square(25) = 26.5543 [0.3785]

  Comparison of information criteria:
  Lag order 2: AIC = 27.1441, BIC = 29.4428, HQC = 27.9812
  Lag order 1: AIC = 26.5723, BIC = 27.8261, HQC = 27.0288
```

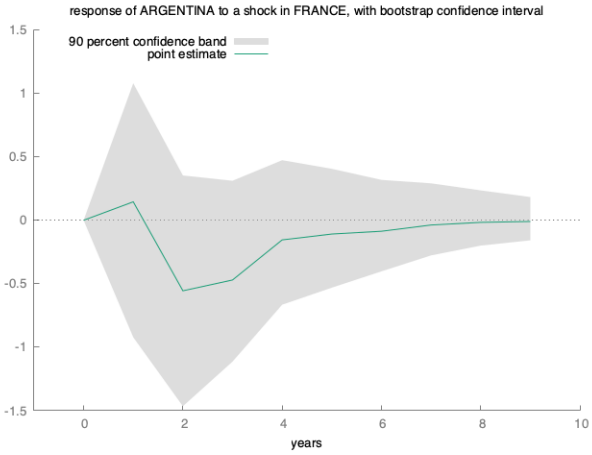
Impulse Response Graphs

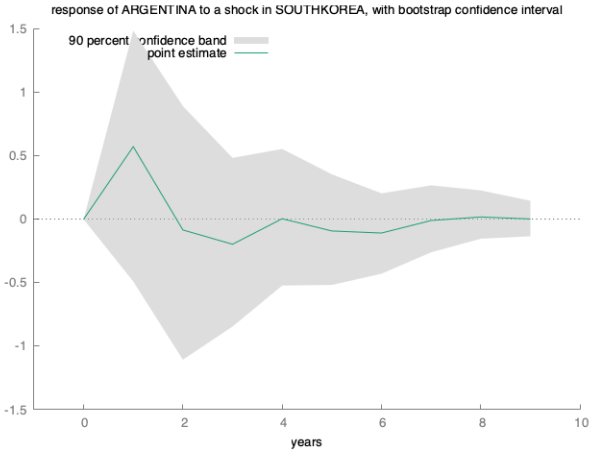
Gretl

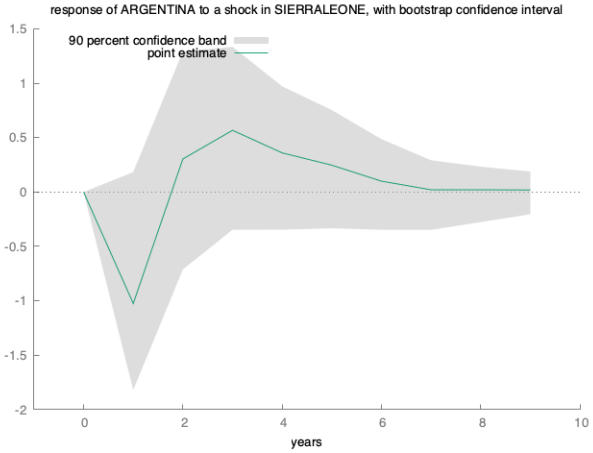
Model → Multivariate Time Series → Vector Autoregression → Select endogenous variables (here, Argentina, France, Sierra Leone, South Korea, USA) → Green Arrow → Select Lag Order (here, 2) → Ok → Graphs → Response of "X" (where "X" represents the endogenous variable of choice) → to "Y" (where "Y" represents the variable "X" responds to) → Select bootstrap confidence interval → Ok

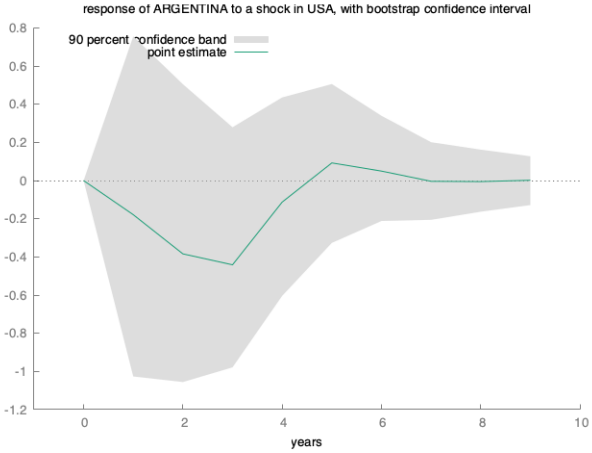
Response of Argentina



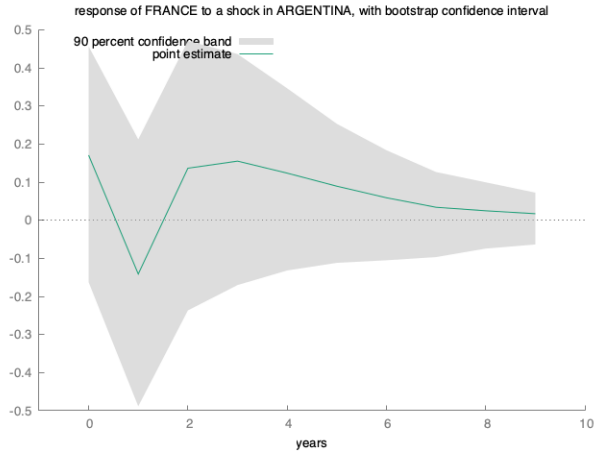


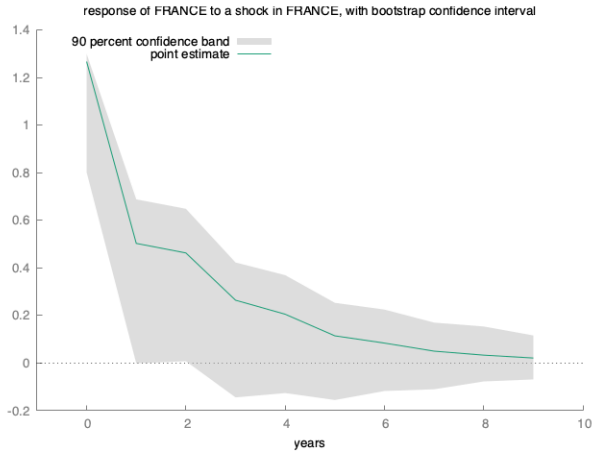


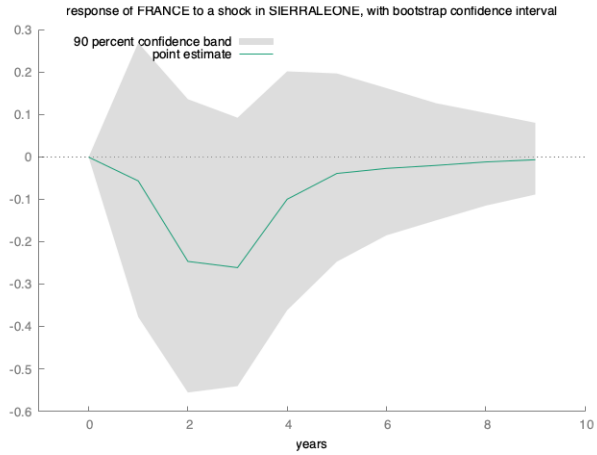


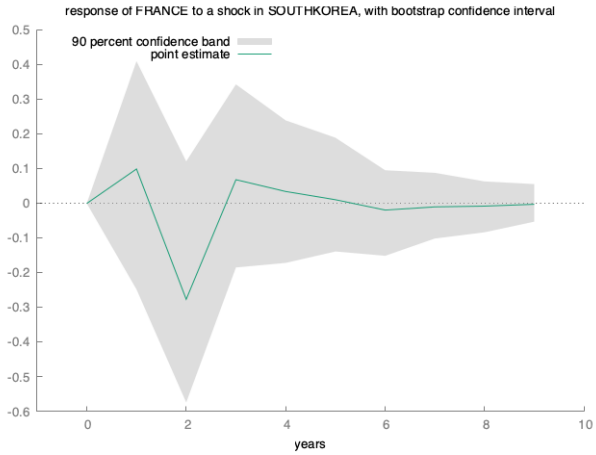


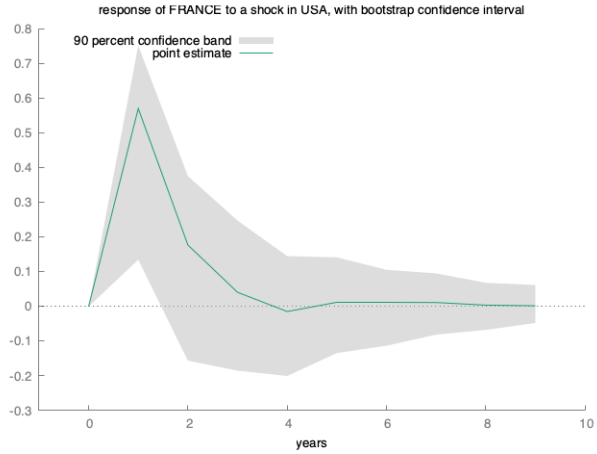
Response of France



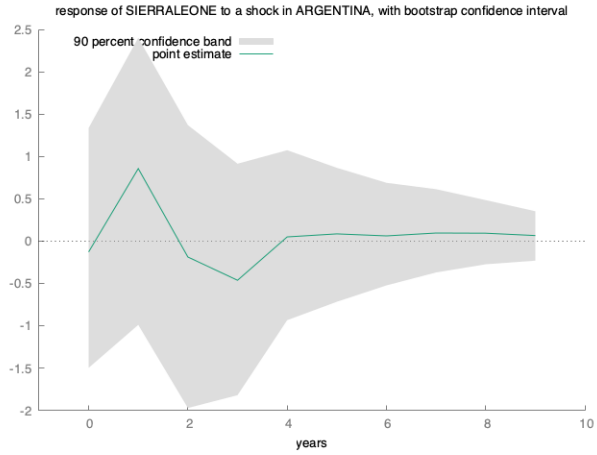


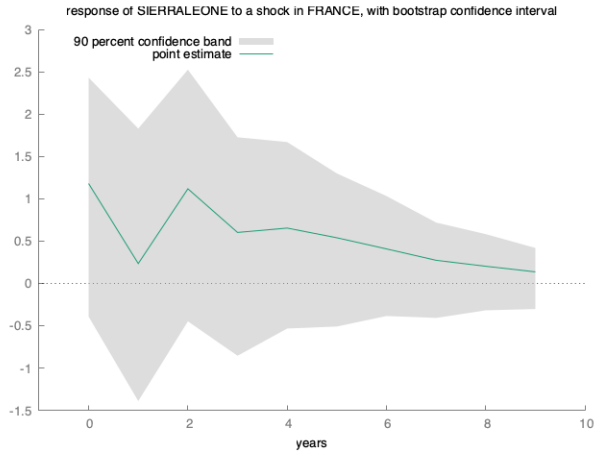


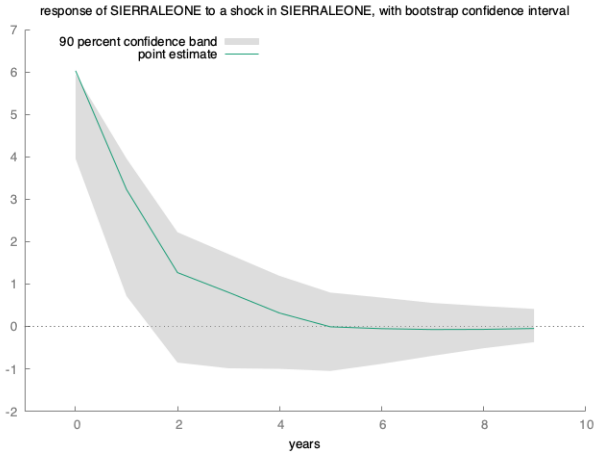


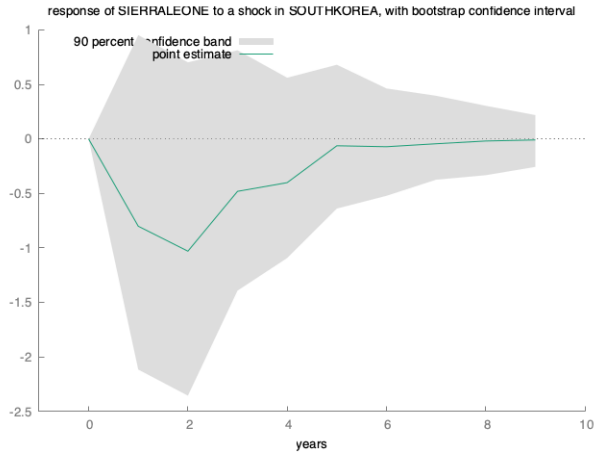


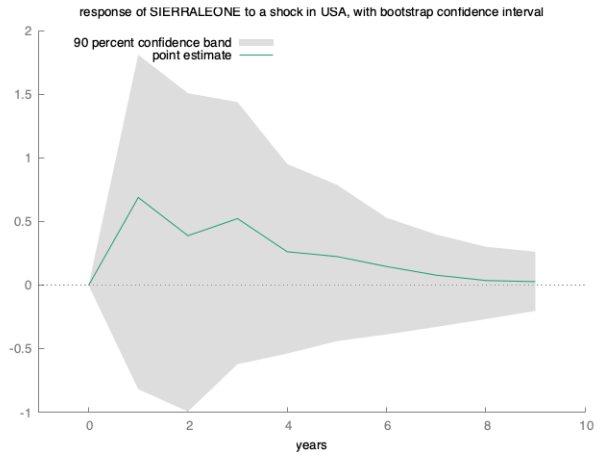
Response of Sierra Leone



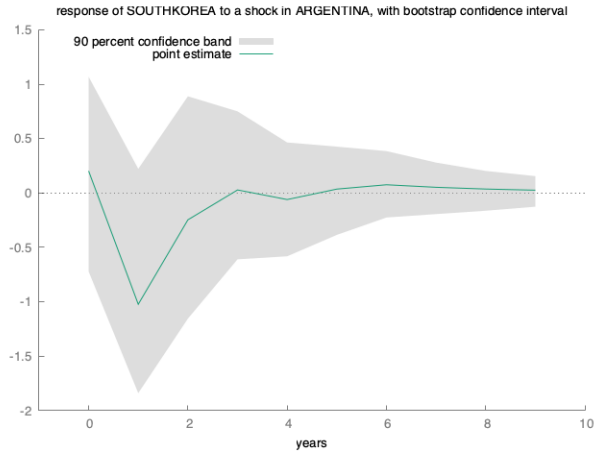


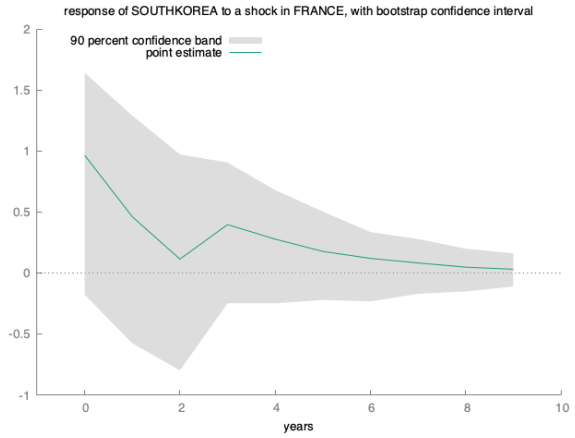


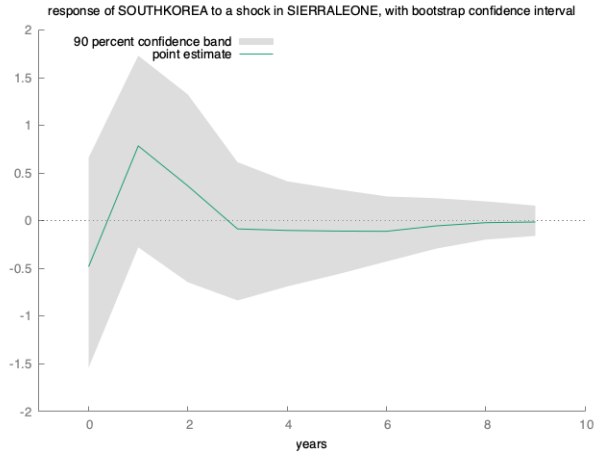


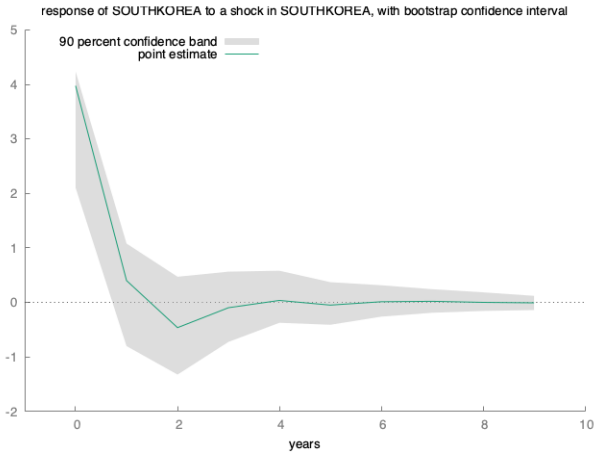


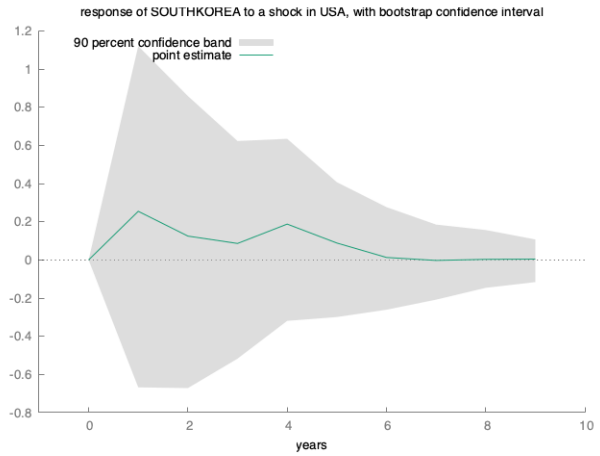
Response of South Korea











Response of USA