Economics 618B: Time Series Analysis
State University of New York at Binghamton
Department of Economics
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Midterm – Answers

1. (a) $E(Y_t) = 0$
   (b) $V(Y_t) = \frac{(q+1)}{(q+1)^2}\sigma^2 = \frac{1}{q+1}\sigma^2$
   (c) $\rho(j) = \begin{cases} (q+1-j)/(q+1) & \text{for } j = 0, 1, 2, \ldots, q \\ 0 & \text{for } j > q \end{cases}$

2. (a) $\widehat{Y}_{t+h|t} = \phi^h Y_t$
   (b) $e_h = Y_{t+h} - \phi^h Y_t = \varepsilon_{t+h} + \phi\varepsilon_{t+h-1} + \phi^2\varepsilon_{t+h-2} + \cdots + \phi^{h-1}\varepsilon_{t+1}$
   (c) $V(e_h) = \sigma^2 \left(1 + \phi^2 + \phi^4 + \cdots + \phi^{2(h-1)}\right)$

3. (a) $E(Y_t) = 0 \ \forall t,\ E\left(Y_t^2\right) = \sigma^2\left(1 - 1.1^2 + 0.18^2\right) \ \forall t,\ E(Y_t Y_{t-1}) = \sigma^2(-1.1 - 1.1 * 0.18),\ E(Y_t Y_{t-2}) = \sigma^2(0.18),\ E(Y_t Y_{t-j}) = 0 \ \forall j > 2$
   (b) The easiest way to show this is to show that the eigenvalues are both inside the unit circle. Rewrite the model as $Y_t = \left(1 - 1.1L + 0.18L^2\right)\varepsilon_t$. Then we can obtain $1 - 1.1z + 0.18z^2 = 0$ or $(z - 0.2)(z - 0.9)$. Note that both the eigenvalues $(0.2, 0.9)$ are inside the unit circle. Alternatively, one can check all of the conditions for stability of the system.

4. (a) ARMA((1,12,13),(0,4))
   (b) $\widehat{Y}_{t+1|t} = Y_{t-11} + \phi(Y_t - Y_{t-12}) + \theta\varepsilon_{t-11}$, $\widehat{Y}_{t+2|t} = Y_{t-10} + \phi(Y_{t+1} - Y_{t-11}) + \theta\varepsilon_{t-10} = Y_{t-10} + \phi^2(Y_t - Y_{t-11}) + \phi\theta\varepsilon_{t-11} + \theta\varepsilon_{t-10}$
   (c) 1 is a unit root, ϕ shows equal, but opposite effects for the other two lags of Y, θ is for the twelves period lag of the error. This is likely a monthly data set.