Economics 618B: Time Series Analysis
Department of Economics
State University of New York at Binghamton
Fall, 2011

Midterm – Answers

The exam is worth 100 points. Each question is of equal value.

1. $E(X_t) = E\left[(-1)^t X\right] = (-1)^t E(X) = 0, \forall t$; $E\left(X_t^2\right) = E\left[(-1)^{2t} X^2\right] = (-1)^{2t} E\left(X^2\right) = \sigma^2, \forall t$; $E(X_t X_{t-j}) = E\left[(-1)^t X (-1)^{t-j} X\right] = (-1)^{2t-j} E\left(X^2\right) = -\sigma^2$ for odd j and σ^2 for even j, $\forall t$
2. $Y_t = \phi Y_{t-1} + \varepsilon_t$
   (a) $\widehat{Y}_{t+h|t} = \phi^h Y_t$
   (b) $e_{t+h} = Y_{t+h} - \widehat{Y}_{t+h|t} = \varepsilon_{t+h} + \phi\varepsilon_{t+h-1} + \cdots + \phi^{(h-1)}\varepsilon_{t+1}$
   (c) $V(e_{t+h}) = \sigma^2\left(1 + \phi^2 + \cdots + \phi^{2(h-1)}\right)$
   (d) $\widehat{Y}_{t+h|t} \pm 1.96\sqrt{V(e_{t+h})}$
3. $y_t = \gamma y_{t-1} + \rho\varepsilon_{t-1} + u_t$, noting that u_t is normally distributed our log-likelihood function is given as $\ln L(\theta) = -\frac{(T-1)}{2}\ln 2\pi - \frac{(T-1)}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=2}^{T}(y_t - \gamma y_{t-1} - \rho\varepsilon_{t-1})^2$ and the FOC are $\frac{\partial}{\partial\gamma} = \frac{1}{\sigma^2}\sum_{t=2}^{T}(y_t - \gamma y_{t-1} - \rho\varepsilon_{t-1})\, y_{t-1}$, $\frac{\partial}{\partial\rho} = \frac{1}{\sigma^2}\sum_{t=2}^{T}(y_t - \gamma y_{t-1} - \rho\varepsilon_{t-1})\,\varepsilon_{t-1}$ and $\frac{\partial}{\partial\sigma^2} = -\frac{(T-1)}{2\sigma^2} - \frac{1}{2\sigma^4}\sum_{t=2}^{T}(y_t - \gamma y_{t-1} - \rho\varepsilon_{t-1})^2$