**3.*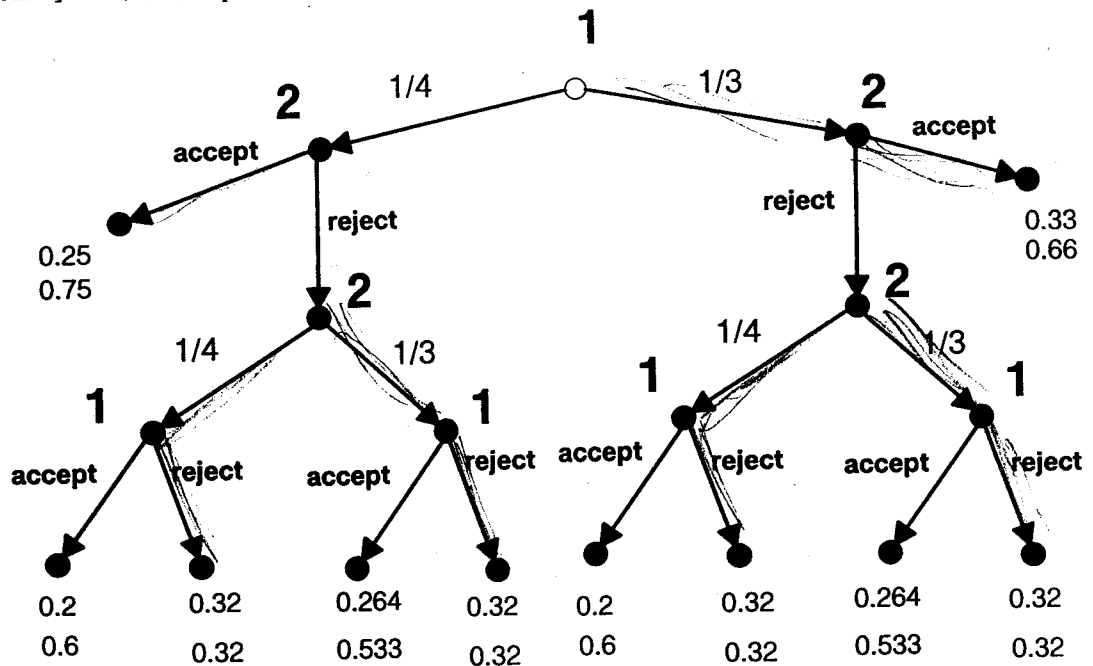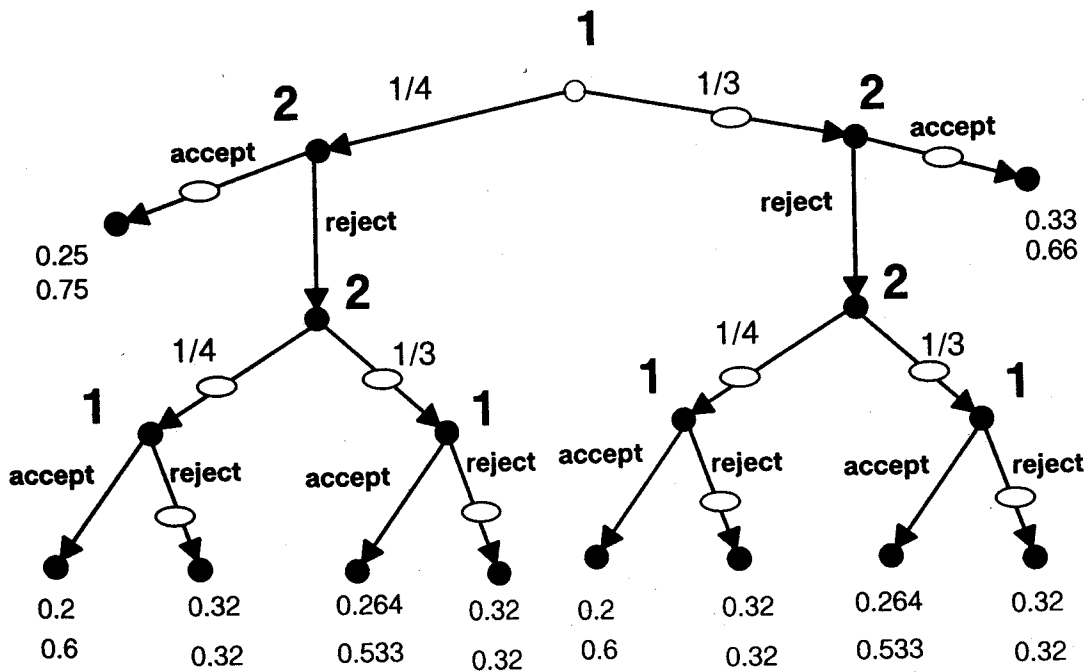* [35 points] Player 1 and Player 2 bargain over one dollar. They take turns making offers: first Player 1 makes a proposal that Player 2 can accept or reject; if 2 rejects then 2 makes a proposal that 1 can accept or reject and then the game ends. Each offer takes one period, and the players are impatient: they discount payoffs received in later periods by the factor d per period (where $0 < d < 1$), so that x received in period 1 are worth x but x received in period 2 are worth only xd and x received in period 3 are worth xd^2. Here is a more detailed description of the game.

(1a)   At the beginning of the first period, Player 1 proposes to take a share s_1 of the dollar, leaving $(1 - s_1)$ for Player 2, where s_1 can be either $\frac{1}{4}$ or $\frac{1}{3}$.

(1b)   Player 2 either accepts the offer – in which case the game ends with Player 1 receiving s_1 and Player 2 receiving $(1 - s_1)$ – or rejects the offer – in which case play continues to the second period.

(2a)   At the beginning of the second period (if any), Player 2 proposes that Player 1 take a share s_2 of the dollar, leaving $(1 - s_2)$ for Player 2, where again s_2 can be either $\frac{1}{4}$ or $\frac{1}{3}$.

(2b)   Player 1 either accepts the offer – in which case the game ends with Player 1 receiving s_2 and Player 2 receiving $(1 - s_2)$ – or rejects the offer – in which case the game proceeds to the third period.

(3)   At the beginning of the third period (if any), each player receives $\frac{1}{2}$.

(a)   Represent this game as an extensive game with perfect information, assuming that $d = 0.8$. [20 points]

(b)   Find the backward induction solution. [10 points]

(c)   Write down one of the many possible strategies for Player 1 [before you answer this question pause for a moment and remind yourself of the definition of strategy for an extensive game]. [5 points]

3. [35 points]   **(a)** [20 points]

1

2 1/4   1/3 **2**
accept   accept

reject   reject

0.25   0.33
0.75   0.66

2

1/4   1/3   1/4   1/3

1   **1**   **1**   **1**

accept reject accept reject accept reject accept reject

0.2   0.32   0.264   0.32   0.2   0.32   0.264   0.32
0.6   0.32   0.533   0.32   0.6   0.32   0.533   0.32

(b) **[10 points]** The backward-induction solution is shown by ovals marking the best choice(s) for each player at every node. Thus Player 1 will offer 1/3 and Player 2 will accept.

1

2 1/4   1/3 **2**
accept   accept

reject   reject

0.25   0.33
0.75   0.66

2

1/4   1/3   1/4   1/3

1   **1**   **1**   **1**

accept reject accept reject accept reject accept reject

0.2   0.32   0.264   0.32   0.2   0.32   0.264   0.32
0.6   0.32   0.533   0.32   0.6   0.32   0.533   0.32

(c) [5 points] Player 1 has five decision nodes hence a strategy must consist of 5 instructions. A possible strategy is: offer ¼, if offered ¼ and 2 rejected and couteroffered ¼ accept, if offered ¼ and 2 rejected and couteroffered 1/3 reject, if offered 1/3 and 2 rejected and couteroffered ¼ reject, if offered 1/3 and 2 rejected and couteroffered 1/3 accept.