Economics 500: Microeconomic Theory

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Department of Economics

Problem Set #8 – Answers

1. John's Lawn Mowing Service is a small business that acts as a price taker (MR = P).The prevailing market price of lawn mowing is $20 per acre. John's costs are given by

$$TC = 0.1q^2 + 10q + 50$$

where *q* is the number of acres John chooses to cut a day.

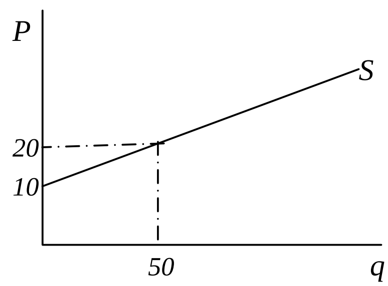
a. How many acres should John choose to cut in order to maximize profit?

$MC=\frac{\partial TC}{\partial q}=0.2q+10$, *set* $MC=P=20 \Rightarrow q^*=50$.

b. Calculate John's maximum daily profit.

$\pi = Pq-TC=1000-800=200$

c. Graph these results and label John's supply curve.



2. Would a lump-sum tax affect the profit-maximizing quantity of output? How about a proportional tax on profits? How about a tax assessed on each unit of output?

$\pi(q)=TR(q)-TC(q)$ *with a lump sum tax T*

$\pi(q)=TR(q)-TC(q)-T \quad \frac{\partial \pi}{\partial q}=\frac{\partial TR}{\partial q}-\frac{\partial TC}{\partial q}-0=0 \Rightarrow MR=MC$, *no change.*

Proportional tax $\pi(q)=(1-t)(TR(q)-TC(q))$

$\frac{\partial \pi}{\partial q}=(1-t)(MR-MC)=0 \Rightarrow MR=MC$, *no change*

Tax per unit $\pi(q)= TR(q)-TC(q)-tq$

$\frac{\partial \pi}{\partial q}=MR-MC-t=0$, *so* $MR=MC+t$, *q is changed* $\Rightarrow$ *A per unit tax does affect output.*

3. A firm faces a demand curve given by

$$q = 100 - 2p$$

Marginal and average costs for the firm are constant at $10 per unit.

a. What output level should the firm produce to maximize profits? What are profits at that output level?

$p=(100-q)/2=50-q/2 \Rightarrow TR=pq=-q^2/2+50q \Rightarrow MR=-q+50$

$MR=MC=10 \Rightarrow q=40,\ \pi=TR-TC=-40^2/2+50*40-10*40=800$

b. What output level should the firm produce to maximize revenues? What are profits at that output level?

$TR=-q^2/2+50q$

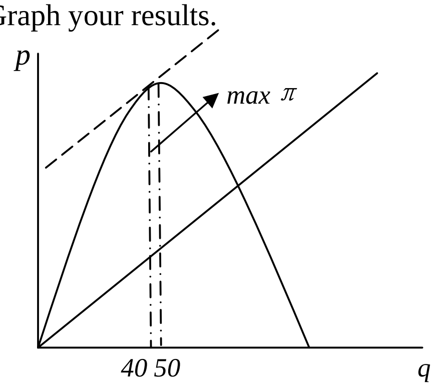
$\frac{\partial TR}{\partial q}=-q+50 \Rightarrow q=50,\quad \pi=TR-TC=-50^2/2+50*50-10*50=750$

c. Suppose the firm wishes to maximize revenues subject to the constraint that it earns $12 in profits for each of the 64 machines it employs. What level of output should it produce?

$\pi=TR-TC=-q^2/2+40q=12*64=768 \Rightarrow q=32\ TR=1088$ or $q=48\ TR=1248.$

$\Rightarrow$*It should produce 48 units.*

d. Graph your results.



4. The production function for a firm in the business of calculator assembly is given by

$$q = 2(L)^{1/2}$$

where q is finished calculator output and L represents hours of labor input. The firm is a price taker for both calculators (which sell for P) and workers (which can be hired at a wage rate of w per hour).

a. What is the supply function for assembled calculators (q = q(p,w))?

$q = 2(L)^{1/2} \Rightarrow L=q^2/4 \Rightarrow TC=Lw=q^2w/4 \Rightarrow MC=qw/2=p \Rightarrow q=2p/w$

b. Show explicitly how changes in w shift the supply curve for this firm.

$p=qw/2 \qquad \frac{\partial p}{\partial w}=q/2>0 \Rightarrow w\uparrow \longrightarrow p\uparrow$