Economics 500: Microeconomic Theory

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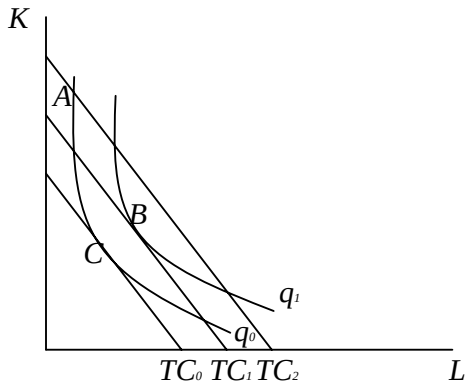
Department of Economics

Problem Set #7 – Answers

1. Explain why the assumption of cost minimization implies that the total cost curve must have a positive slope: An increase in output must always increase total cost.
   The assumption of cost minimization is that the firm will minimizes the cost of any given level of output.
   Suppose the firm produces q_0 at C, which is the cost minimizing point.
   When the firm increases output to q_1, the isoquant will shift from q_0 to q_1 and the total cost must increase (from C to B).
   Otherwise, if the output increases while the total cost decreasing from A to B, then it suggests that at q_0, the firm dose not minimize cost, in other words, the firm can produce more output with less cost (B).
   If an increase in output can decrease total cost, the production of the firm will explode to infinity.
   Therefore, an increase in output must always increase total cost (marginal cost>0)

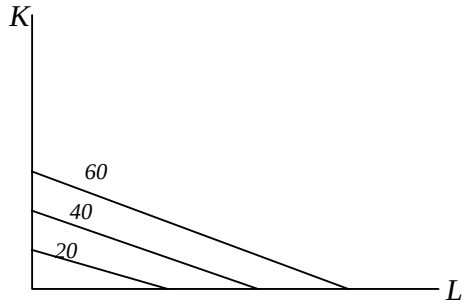


2. A firm purchases capital and labor in competitive markets at prices of r = 6 and w = 4, respectively. With the firm's current input mix, the marginal product of capital is 12 and the marginal product of labor is 18. Is this firm minimizing its costs? If so, explain how you know. If not, explain what the firm ought to do.
   No. The necessary condition of cost minimization is: $MP_K/MP_L=r/w$. Here, $MP_K/MP_L=12/18=2/3$, $r/w=3/2$ ➔ $MP_K/MP_L<r/w$ ➔The firm does not minimize its cost.
   Given perfect competitive condition and r,w constant, according to diminishing marginal productivity, the firm should decrease the input of capital and increase the input of labor until $MP_K/MP_L=3/2$.

3. A widget manufacturer has an infinitely substitutable production function of the form

$$q = 2K + L$$

a. Graph the isoquant maps for q = 20, q = 40, and q = 60. What is the MRTS along these isoquants?



q=20 ➔K=-L/2+10
q=40 ➔K=-L/2+20
q=60 ➔K=-L/2+30
MRTS=1/2 ➔The three isoquants has the same MRTS.

b. If the wage rate (w) is $1 and the rental rate on capital (r) is $1, what costminimizing combination of K and L will the manufacturer employ for the three different production levels in part (a)?
$MP_K/MP_L=2>r/w=1$, since here MRTS and r/w are constant ➔the manufacturer should choose the relative cheaper one (K).
q=20 ➔L=0,K=10; q=40 ➔L=0,K=20; q=60 ➔L=0,K=30

c. How would your answer change to part (b) if r rose to $3 with w remaining at $1?
The same reasoning as part b, he will choose the relative cheaper one (L).
q=20 ➔L=20,K=0; q=40 ➔L=40,K=0; q=60 ➔L=60,K=0

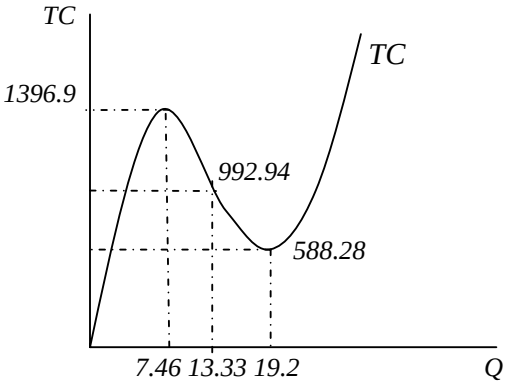
4. A stuffed-wombat manufacturer determined that the lowest average production costs were achieved when eight wombats were produced at an average cost of $1,000 each. If the marginal cost curve is a straight line intersecting the origin, what is the marginal cost of producing the ninth wombat?
At TC minimum point, AC=MC, AC(8)=1000 ➔MC(8)=1000
MC(0)=0, since MC curve intersects the origin.
➔MC function: MC=1000q/8 =125q ➔when q=9, MC=1125.

5. The long-run total cost function for a firm producing skateboards is

$$TC = q^3 - 40q^2 + 430q$$

where q is the number of skateboards per week.

a. What is the general shape of this total cost function?



When $q<\frac{40-\sqrt{310}}{3}$ *=7.46, TC'(q)>0, TC is increasing.*

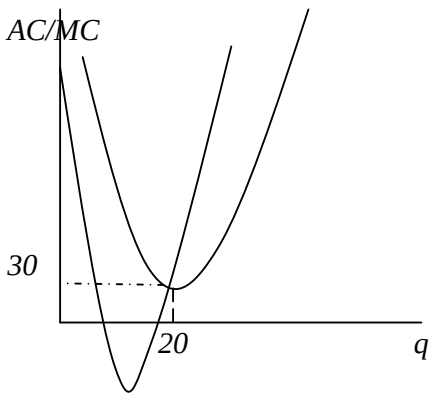
When $q>\frac{40-\sqrt{310}}{3}$ *=19.2,TC'(q)>0, TC is increasing.*

When 7.46<q<19.2, TC'(q)<0, TC is decreasing.
When q>=40/3=13.33, TC''(q)>=0, TC curve is convex.
When q<=13.33, TC''(q)<=0, TC curve is concave.

b. Calculate the average cost function for skateboards. What shape does the graph of this function have? At what level of skateboard output does average cost reach a minimum? What is the average cost at this level of output?
AC=TC/q=q^2 *-40q+430, AC'=2q-40=0, AC''=2>0* ➔ *q=20, AC=30, minimum.*

c. Determine the marginal cost for skateboards, show that this marginal cost curve intersection average cost at its minimum value.
MC=$3q^2$ *-80q+430,*
Set MC=AC, $3q^2$ *-80q+430=*q^2 *-40q+430* ➔ *q=20 or q=0, at q=20 AC is minimum.*

d. Graph the average and marginal cost curves for skateboard production.



6. Professor Smith and Professor Jones are going to produce a new introductory textbook. As true scientists, they have laid out the production function for the book as

$$q = S^{1/2} J^{1/2}$$

where q is the number of pages in the finished book, S is the number of working hour spent by Smith and J is the number of hours spent working by Jones. Smith values his labor as $3 per working hour. He has spent 900 hours preparing the first draft. Jones, whose labor is valued at $12 per working hour, will revise Smith's draft to complete the book.

a. How many hours will Jones have to spend to produce a finished book of 150 pages? Of 300 pages? Of 450 pages?

$q=900^{1/2} J^{1/2}$ ➔$J=q^2/900$

When q=150, J=25; when q=300, J=100; when q=450, J=225

b. What is the marginal cost of the 150th page of the finished book? Of the 300th page? Of the 450th page?

$TC=w_sS+w_jJ=2700+12q^2/900=q^2/75+2700$ ➔$MC=2q/75$

When q=150, MC=4; when q=300, MC=8; when q=450, J=12

7. A firm producing hockey sticks has a production function given by

$$q = 2(KL)^{1/2}$$

In the short run, the firm's amount of capital equipment is fixed at K = 100. The rental rate for K is r = $1, and the wage rate for L is w = $4.

a. Calculate the firm's short-run total cost curve. Calculate the short-run average cost curve.

SR: $q=2(100L)^{1/2}$ ➔$L=q^2/400$

$SRTC=Kr+Lw=100+4q^2/400=100+q^2/100$

$SRATC=TC/q=100/q+q/100$

b. What is the firm's short-run marginal cost function? What are the SRTC, SRATC and SRMC for the firm if it produces 25 hockey sticks? Fifty hockey sticks? One hundered hockey sticks?

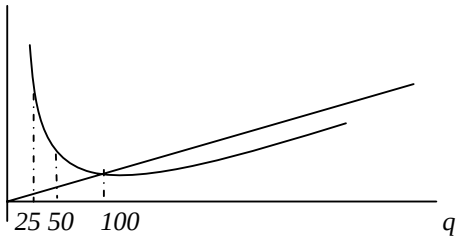
$SRMC=q/50$

$q=25$, $SRTC=100+25^2/100=106.25$, $SRATC=4.25$, $SRMC=0.5$

$q=50$, $SRTC=125$, $SRATC=2.5$, $SRMC=1$

$q=100$, $SRTC=200$, $SRATC=2$, $SRMC=2$

c. Graph the SRATC and the SRMC curves for the firm. Indicate the points found in part (b).*SRATC/SRMC*



d. Where does the SRMC curve intersect the SRATC curve? Explain why the SRMC curve will always intersect the SRATC at its lowest point.
SRMC curve intersects SRATC curve at the minimum of SRATC, because:
When SRMC<SRATC, SRATC is decreasing.
When SRMC>SRATC, SRATC is increasing.
When SRMC=SRATC, SRATC is at the stationary point, which is the minimum.