Economics 500: Microeconomic Theory

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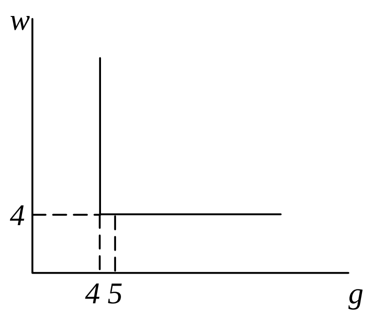
Problem Set #6 – Answers

1. Roy Dingbat is the manager of a hot dog stand that uses only labor and capital to produce hot dogs. The firm usually produces 1,000 hot dogs a day with 5 workers and 4 grills. One day a worker is absent but the stand still produces 1,000 hot dogs. What does this imply about the 1,000 hot dog isoquant? What does this imply about Roy's management skills?

This suggests the 1000 hot dog isoquant is L-shaped (Lieontief).



This implies Roy's management skill is not good, because he does not minimize the production cost of 1000 hot dogs.

2. Marjorie Cplus wrote the following answer on her micro examination: "Virtually every production function exhibits diminishing returns to scale because my professor said that all inputs have diminishing marginal productivities. So when all inputs are doubled, output must be less than double." How would you grade Marjorie's answer?

His answer is wrong. Since "diminishing returns to scale" and "diminishing marginal productivities" are under different assumptions.

DRS assumes that if all inputs increase by the same constant number λ *(>1), then output increases by less than* λ.

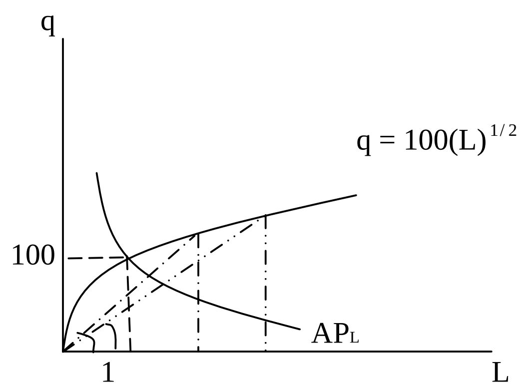
The diminishing MP assumes that additional output decrease by employing one more unit of input, while holding all other inputs constant.

3. Digging clams by hand in Sunset Bay requires only labor input. The total number of clams obtained per hour (q) is given by

$$q = 100(L)^{1/2}$$

where L is labor input per hour.

a. Graph the relationship between q and L.



b. What is the average productivity of labor in Sunset Bay? Graph this relationship and show that AP_L diminishes for increases in labor input.

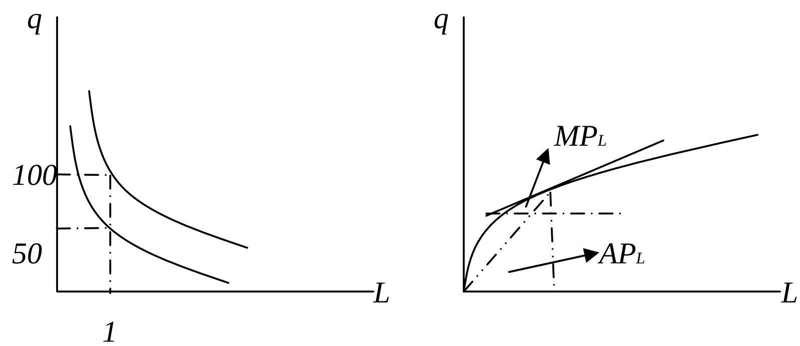
$AP_L = q/L = 100(L)^{-1/2}$ see the above graph, we can see the diminishing relationship through the AP_L curve or through the production curve directly, where AP_L is the slope of the dotted line between the origin and the production curve.

Mathematically, $\frac{dAP_L}{dL} = 100(-1/2)L^{-3/2} = -50L^{-3/2} < 0$

➔AP_L diminishes for increases in labor input.

c. Determine the marginal productivity of labor. Graph this relationship and show that $MP_L < AP_L$ for all values of L. Explain why this is so.

$MP_L = \frac{dq}{dL} = 50L^{-1/2}$



We can see from the graph $MP_L < AP_L$ for all values of L.
Mathematically, $AP_L / MP_L = 2 > 1$ ➔ $MP_L < AP_L$ for all values of L.
$MP_L < AP_L$, because this production function exhibits diminishing marginal productivity of labor.

4. Suppose the production function for widgets is given by

$$q = KL - 0.8K^2 - 0.2L^2$$

where q represents the annual quantity of widgets produced, K represents annual capital input, and L represents annual labor input.

a. Suppose K = 10; graph the total and average productivity of labor curves. At what level of labor input does this average productivity reach a maximum? How many widgets are produced at that point?

When K=10, $10L-0.2L^2$ -80q=10L-80-0.2L^2

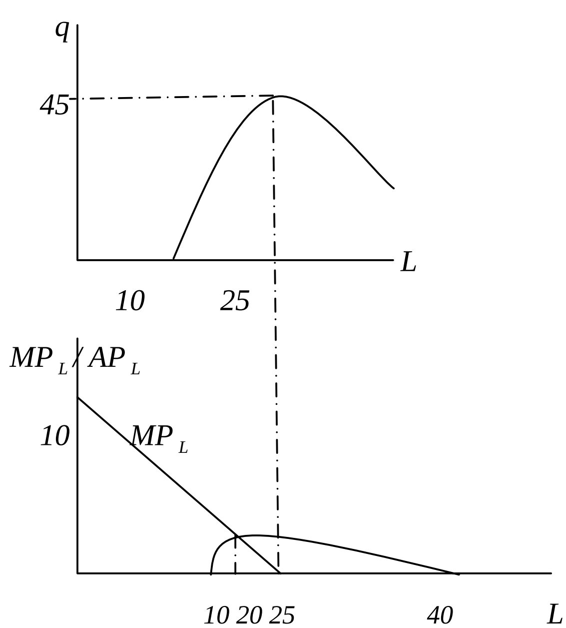
MP_L =dq/dL=10-0.4L=0 ➔max at L=25

$$\frac{d^2q}{dl^2} = -0.4$$ *, the total product curve is concave.*

AP_L =q/L=10-0.2L-80/L

To graph this curve: $\frac{dAP_L}{dL} = \frac{80}{L} - 0.2 = 0$ ➔*max at L=20.*

When L=20, q=40, AP_L =0



b. Again assuming that K = 10, graph the MP_L curve. At what level of labor input does MP_L = 0?

MP_L =10-0.4L=0➔L=25, see above graph.

c. Does the widget production function exhibit constant, increasing or decreasing returns to scale?

Suppose K, L increasing by λ (>1)

$q_1 = \lambda K \lambda L - 0.8(\lambda K)^2 - 0.2(\lambda L)^2 = \lambda^2 KL - \lambda^2 0.8K^2 - \lambda^2 0.2L^2 = \lambda^2 q$

➔ $\lambda > 1$ ➔ $\lambda^2 > \lambda$ ➔ $\lambda^2 q > \lambda q$ ➔ $q_1 > \lambda q$

➔*The function exhibits increasing returns to scale.*

5. Suppose that

$$q = L^a K^b,$$

where $0 < a < 1$, $0 < b < 1$, $a + b = 1$.

a. Show that $MP_L > 0$, $MP_K > 0$, and that the second partials are negative.

$MP_L = \frac{\partial q}{\partial L} = aL^{a-1}K^b$ ➔ $MP_L > 0$, *since* $0<a<1$.

$\frac{\partial MP_L}{\partial L} = a(a-1)L^{a-2}K^b < 0$, *since* $0<a<1, a-1<0$.

Similarly, $MP_K > 0$, $\frac{\partial MP_K}{\partial K} < 0$

b. Show that the MRTS depends only on K/L, but not on the scale of production, and that the MRTS diminishes as L/K increases.

$MRTS = \frac{MP_L}{MP_K} = \frac{K}{L}\frac{a}{b}$, *MRTS depends on the ratio of K and L, not the amount of K and L, that is the scale of production.*

$\frac{\partial MRTS}{\partial \frac{L}{K}} = -1(\frac{L}{K})^{-2}\frac{a}{b} < 0$ ➔*MRTS diminishes as L/K increases.*