Economics 500: Microeconomic Theory

State University of New York at Binghamton
Department of Economics
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Problem Set #2

1. Laidback Al derives utility from 3 goods: music (M), wine (W) and cheese (C). His utility function is of the simple linear form

$$U(M,W,C) = M + 2W + c$$

   a. Assuming consumption of music is fixed at 10, determine the equations for the indifference curves for W and C for U = 40 and U = 70. Sketch these curves.
   b. Show that Al's MRS of wine for cheese is constant for all values of W and C on the indifference curves for calculated in part (a).
   c. Suppose Al's consumption of music increase to 20. How would this change your answers to parts (a) and (b)? Explain your results intuitively.

2. Suppose the utility function for two goods, X and Y, has a Cobb-Douglas form

$$U(X,Y) = (XY)^{1/2}$$

   a. Graph the U = 10 indifference curve associated with this utility function.
   b. If X = 5, what must Y equal to be on the U = 10 indifference curve? What is the MRS at this point?
   c. In general, develop an expression for the MRS for this utility function. Show how this can be interpreted as the ratio of marginal utilities for X and Y.

3. Georgia always eats hot dogs in a bun together with 1 oz. of mustard. Each hot dog eaten this way provides 15 units of utility, but any other combination of hot dogs, buns and mustard is worthless to Georgia.
   a. Explain the nature of Georgia's utility function and indicate the form of her indifference curve map.
   b. Suppose hot dogs cost $1, buns cost $0.40 and mustard costs $0.10 per ounce. Show how Georgia's utility can be represented by the total amount of money she spends on these three items.
   c. How would your answer to part (b) change if the price of hot dogs rose to $1.50?

4. Consider the following Cobb-Douglas utility function

$$U(X,Y) = X^aY^b$$

   a. Calculate the MRS
   b. Does this result depend on whether a + b = 1? Does this sum have any relevance to the theory of choice?
   c. For commodity bundles for which Y = X, how does the MRS depend on the values of a and b? Develop an intuitive explanation of why if a > b, MRS > 1. Illustrate your argument with a graph.

5. Each day Paul, who is in the third grade, eats lunch at school. He likes only Twinkies (T) and Orange Slice (S), and these provide him with a utility of

$$U(T,S) = (TS)^{1/2}$$

   a. If Twinkies cost \$0.10 each and Slice costs \$0.25 per cup, how should Paul spend the \$1.00 his mother gives him in order to maximize his utility?
   b. If the school tries to discourage Twinkie consumption by raising the price to \$0.40, by how much will Paul's mother have to increase his lunch allowance to provide him with the same level of utility he received in part (a)? How many Twinkies and cups of Slice will he buy now (assuming that it is possible to purchase fractional amounts of both of these goods)?

6. A young connoisseur has \$300 to spend to build a small wine collection. She enjoys two vintages in particular: an expensive 1987 French Bordeau (F) at \$20 per bottle and a less expensive 1993 California varietal wine (C) priced at \$4. How much of each wine should she purchase if her utility is characterized by the following function?

$$U(F,C) = F^{2/3}C^{1/3}$$

   When she arrived at the wine store, our young oenologist discovered that the price of the 1987 French Bordeau had fallen to \$10 a bottle because of a decline in the value of the franc. If the price of the California wine remains stable at \$4 per bottle, how much of each wine should our friend purchase to maximize her utility under these altered conditions?

7. On a given evening J.P. enjoys the consumption of cigars (C) and brandy (B) according to the function

$$U(C,B) = 20C - C^2 + 18B - 3B^2$$

   a. How many cigars and glasses of brandy does he consume during an evening (Cost is now object to J.P.)?
   b. Lately, however, J.P. has been advised by his doctors that he should limit the sum of brandy and cigars consumed to 5 per night. How many glasses of brandy and cigars will he consume now?

8. Suppose individuals require a certain level of food (X) to remain alive. Let this amount be given by X_o. Once X_o is purchased, individuals obtain utility from food and other goods (Y) of the form

$$U(X,Y) = (X - X_o)^a Y^b$$

   where $a + b = 1$. Show that if $I > P_x X$ the individual will maximize utility by spending $a(I - P_x X_o) + P_x X_o$ on good X and $b(I - P_x X_o)$ on good Y.