Economics 500: Microeconomic Theory
State University of New York at Binghamton
Department of Economics
Fall, 2004

Problem Set #1

1. For each of the following functions of one variable, determine all local maxima and minima and indicate points of inflection (where $f''(x) = 0$)
   a. $f(x) = 4x^3 - 12x$
   b. $f(x) = 4x - x^2$
   c. $f(x) = x^3$

2. Suppose $U = (x,y) = 4x^2 + 3y^2$.
   a. Calculate the partials of U with respect to x and y
   b. Evaluate these partial derivatives at $x = 1$ and $y = 2$.
   c. Write the total differential for U.
   d. Calculate dy/dx for $dU = 0$ (that is, what is the implied trade-off between x and y holding U constant?).
   e. Show $U = 16$ when $x = 1$ and $y = 2$.
   f. In what ratio must x and y charge to hold U constant at 16 for movements away from $x = 1$ and $y = 2$?
   g. More generally, what is the shape of the $U = 16$ contour line for this function? What is the slope of that line?

3. Suppose $f(x,y) = xy$. Find the maximum value for f if x and y are constrained to sum to 1. Solve this problem in two ways: by substitution and by using the Langrangian multiplier method.

4. Suppose a firm's total revenues depend on the amount produced (q) according to the function
$$TR = 70q - q^2$$
   Total costs also depend on q:
$$TC = q^2 + 30q + 100$$
   a. What level of output should the firm produce in order to maximize profits (TR – TC)? What will profits be?
   b. Show that the second-order conditions for a maximum are satisfied at the output level found in part (a).
   c. Does the solution calculated here obey the "marginal revenue equals marginal cost" rule? Explain.

5. A manufacturer produces gizmos at a cost of $5 each. The manufacture computes that if each gizmos sells for x dollars, $(15 - x)$ gizmos will be sold. What is the manufacturer's profit function? What price should the manufacturer charge to maximize profit?

6. A manufacturer can produce economics texts at a cost of $5 apiece. The text currently sells for $10, and at this price 10 texts are sold each day. The manufacturer figures that each dollar decrease in price will sell one additional copy each day. Write out the demand and profit functions. What price x maximizes profit?

7. Find all the first and second-order partial derivatives of the following functions:
   a. $f(x,y) = ax^2 + bxy + cy^2$
   b. $f(x,y,z) = ax^2 + by^2 + cz^2 + dxy + exz + gyz$
   c. $f(x,y) = (xy)^{1/2}$
   d. $f(x,y,z) = dx^a y^b z^c$
   e. Find the total differential of each of the functions in a-d