Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm II

The exam consists of four questions on four pages. Each question is of equal value.

1. Consider the model $y_t = c + \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$, where $\varepsilon_t \sim WN\left(0, \sigma_\varepsilon^2\right)$ is a white noise sequence.

   (a) Write the log-likelihood function needed to estimate this model.

   (b) Consider the null hypothesis $H_0 : \phi = 0$. Write the log-likelihood function needed to estimate the model under the null hypothesis.

   (c) Write the likelihood ratio test relevant for the test in part (b).

   (d) Consider the null hypothesis $H_0 : \theta = \phi = 0$. Write the log-likelihood function needed to estimate the model under the null hypothesis.

   (e) For the objective function in part (d), derive the estimator of c.

2. Consider two processes Y_t and X_t. Suppose that Y_t represents yearly personal consumption expenditures on food and X_t represents disposable personal income. Suppose we are interested in eventually forecasting Y with the help of both past values of Y as well as past values of X. With this information, answer the following questions

   (a) Write down an ARMA(0,1) which also includes a (single) first lagged value for X

   (b) How do you know how many lags to include for Y? How do you know how many lags to include for ε (the error term)? How do you know how many lags to include for X?

   (c) Write down the h-step ahead value for Y (Y_{t+h}) that you put down in part (a)

   (d) Construct the forecast value of Y ($\widehat{Y}_{t+h|t}$) for $h = 1$

   (e) Why is it difficult to forecast for $h = 2$ and beyond?

3. Suppose the true data generating process is $y_t = c + \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$, but you choose your tentative model to be $y_t = c + \phi y_{t-1} + \varepsilon_t$. Assuming that $|\phi| < 1$, answer the following:

(a) Write both the likelihood function for the true model and for your model.

(b) Draw a sample ACF and PACF for the *residuals* from both the true model and for your model.

(c) What will the null hypothesis and conclusion from the Box and Pierce test (Q-statistic) be for both the true model and for your model?

(d) What is the consequence from choosing the wrong model for point forecasts (for $h = 1$)?

(e) What is the consequence from choosing the wrong model for interval forecasts (for $h = 1$)?

4. Consider the R output listed below. With this information, answer the following:

model.fit = arima(data,order=c(0,0,2),method='ML')
model.fit
Coefficients:

	ma1	ma2	intercept
	0.4705	0.2496	-0.0190
s.e.	0.7376	0.3428	1.7343

sigma2 estimated as 40.51: log likelihood = -140.76, AIC = 291.53, AICc = 293.15, BIC = 300.33

(a) Find the h-step ahead forecast for $h = 1, 2, \ldots$

(b) Find the h-step ahead forecast error for $h = 1, 2, \ldots$

(c) Find the h-step ahead forecast error variance for $h = 1, 2, \ldots$

(d) Find the h-step ahead forecast interval forecast for $h = 1, 2, \ldots$

(e) Plots parts (a) and (d) in a single figure